APPLICATION OF DERIVATIVE

Very short answer type question

1. Find the angle θ, where $0 < \theta < \frac{\pi}{2}$, which increases twice as fast as its sine.
2. Find the slope of the normal to the curve $x = a\cos^3\theta$ and $y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$.
3. A balloon which always remains spherical has a variable radius. Find the rate at which its volume is increasing with respect to its radius when the radius is 7 cm.
4. Write the interval for which the function $f(x) = \cos x$, $0 \le x \le 2\pi$ is decreasing.
5. If the rate of change of area of a circle is equal to the rate of change of its diameter. Find the radius of a circle.
6. For what values of x is the rate of increasing of $x^3 - 5x^2 + 5x + 8$ is twice the rate of increase of x ?
7. Find the point on the curve $y = x^2 - 2x + 3$ where the tangent is parallel to x-axis.
8. Find the maximum value of $f(x) = 2x^3 - 24x + 107$ in the interval $[1, 3]$.
9. Write the maximum value of $f(x) = \frac{logx}{x}$, if it exists.
10. Find the least value of $f(x) = ax + \frac{b}{x}$ where $a > 0, b > 0$ and $x > 0$.
11. The sum of two number is 8, what will the maximum value of the sum of their reciprocals.
12. Find the interval in which the function $f(x) = x - e^x + \tan\left(\frac{2\pi}{7}\right)$ increases.
13. For the curve $y = (2x+1)^3$ find the rate of change of the slope of the tangent.
14. Find the coordinates of the point on the curve $y^2 = 3 - 4x$, where tangent is parallel to the line $2x + y - 2 = 0$.
15. Find the value of a for which the function $f(x) = x^2 - 2ax + 6,\ x > 0$ is strictly increasing.

4 Mark Questions

Rate of change

16. In a competition, a brave child tries to inflate a huge spherical balloon bearing slogans against child labour at the rate of 900cm^3 of gas per second. Find the rate at which the radius of the balloon is increasing, when radius is 5cm. Why is child labour not good for society?
17. An inverted cone has a depth of 10cm and a base radius of 5cm. Water is poured into it at the rate of $\frac{3}{2}cm^3/minute$. Find the rate at which the level of water in the cone is rising when the depth is 4cm.
18. The volume of a cube is increasing at a constant rate. Prove that the increase in its surface area varies inversely as the length of an edge of cube

19. A kite is moving horizontally at a height of 151.5 meters. If the speed of the kite is 10m/sec, how fast is the string being let out when the kite is 250m away from the boy who is flying the kite? The height of the boy is 1.5m.
20. A swimming pool is to be drained for cleaning. If L represents the number of liters of water in the pool t seconds after the pool has been plugged off to drain and $L = 200(10 - t)^2$. How fast is the water running out at the end of 5 sec and what is the average rate at which the water flows out during the first 5 seconds?
21. The sides of an equilateral triangle are increasing at the rate of 2cm/sec. Find the rate at which the area increases, when the side is 10cm.
22. A man 2m tall, walk at uniform speed of 6km/h away from a lamp post 6m high. Find the rate at which the length of his shadow increases.
23. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lower most. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of $5m^3/h$. Find the rate at which the level of water is rising at the instant, when the depth of water in the tank is 4m.
24. A spherical ball of salt is dissolving in water in such a manner that the rate of decrease of the volume at any instant is proportional to the surface area. Prove that the radius is decreasing at a constant rate.
25. A conical vessel whose height is 10 meters and the radius of whose base is half that of the height is being filled with a liquid at a uniform rate of $1.5m^3/min$. Find the rate at which the level of the water in the vessel is rising when it is 3m below the top of the vessel.
26. $x \text{ and } y$ are the sides of two squares such that $y = x - x^2$. Find the rate of change of area of the second square w.r.t the area of the first square.
27. The length of a rectangle is increasing at the rate of 3.5cm/sec. and its breadth is decreasing at the rate of 3cm/sec. Find the rate of change of the area of the rectangle when length is 12cm and breadth is 8cm.
28. If the areas of a circle increases at a uniform rate, then prove that the perimeter various inversely as the radius.

Increasing and Decreasing

29. Show that $f(x) = x^3 - 6x^2 + 18x + 5$ is an increasing function for all $x \in R$. Find the its value when the rate of increase of $f(x)$ is least.
30. Determine whether the following function is increasing or decreasing in the given interval: $f(x) = \cos\left(2x + \frac{\pi}{4}\right),\ \frac{3\pi}{8} \le x \le \frac{5\pi}{8}$.
31. Determine for which values of x, the function $y = x^4 - \frac{4x^3}{3}$ is increasing and for which it is decreasing.

32.Find the interval of increasing and decreasing of the function $f(x) = \frac{logx}{x}$

33.Find interval of increasing and decreasing of the function $f(x) = \sin x - \cos x\ 0 < x < 2\pi$.

34.Show that $f(x) = x^2e^{-x}$, $0 \le x \le 2$ is increasing in the indicated interval.

35.Prove that the function $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$.

36.Find the intervals in which the following function is decreasing.

$$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$$

37.Find the intervals in which the function $f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}, x > 0$ is strictly decreasing.

38.Show that the function $f(x) = \tan^{-1}(\sin x + \cos x)$, is strictly increasing in the interval $\left(0, \frac{\pi}{4}\right)$.

39.Find the interval in which the function $f(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is increasing or decreasing.

40.Find the interval in which the function given by $f(x) = \frac{3x^4}{10} - \frac{4x^3}{5} - 3x^2 + \frac{36x}{5} + 11$. is strictly increasing and strictly decreasing.

Tangent and Normal

41.Find the equation of the tangent to the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(\sqrt{2}a, b)$.

42.Find the equation of the tangent to the curve $y = x^2 - 2x + 7$ which is (i) Parallel to the line $2x - y + 9 = 0$ (ii) Perpendicular to the line $5y - 15x = 13$

43.Find the coordinates of the point on the curve $\sqrt{x} + \sqrt{y} = 4$ at which tangent is equally inclined to the axis.

44.Find the equation of the normal to the curve $y = e^{2x} + x^2$ at $x = 0$. also find the distance from origin to the line.

45.Show that the line $\frac{x}{a} + \frac{y}{b} = 1$ touches the curve $y = be^{\frac{-x}{a}}$ at the point where the curve intersects the axis of y.

46.Show that the equation of normal at any point θ on the curve $x = 3\cos\theta - \cos^3\theta$, $y = 3\sin\theta - \sin^3\theta$ is $4(y\cos^3\theta - x\sin^3\theta) = 3\sin 4\theta$.

47.Show that the curves $xy = a^2$ and $x^2 + y^2 = 2a^2$ touch each other.

48.For the curve $y = 5x - 2x^3$ if x increases at the rate of 2 units/sec then how fast the slope of the curve changing when x=3?

49.Find the condition for the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $xy = c^2$ to intersect orthogonally.

50.Show that the curves $y = a^x \ and \ y = b^x, a > b > 0$ intersect at an angle of $\tan^{-1}\left(\left|\frac{log\left|\frac{a}{b}\right|}{1+loga\ logb}\right|\right)$

51.Find the equation of the normal at a point on the curve $x^2 = 4y$ which passes through the point $(1, 2)$. Also find the equation of the corresponding tangent.

52.Find the point on the curve $9y^2 = x^3$ where the normal to the curve makes equal intercepts with the axis.

Approximation

53.Find the value of $log_{10}(10.1)$given that $log_{10}e = 0.4343$.

54.If the radius of a circle increases from 5cm to 5.1cm, find the increase in the area.

55.If sides of a cube be increased by 0.1%, find the corresponding increase in the volume of the cube.

56.Find the approximate value of $\frac{1}{\sqrt{25.1}}$, using differentials.

57.The radius of a sphere shrinks from 10cm to 9.8cm. Find the approximate decrease in its volume.

Maxima and Minima

(6 Marks Question)

58.Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6\sqrt{3}$r.

59.If the sum of length of hypotenuse and a side of a right angled triangle is given, show that the area of a triangle is maximum, when the angle between them is $\frac{\pi}{3}$.

60.Show that semi-vertical angle of a cone of maximum volume and given slant height is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

61.The sum of the surface areas of a cuboids with sides $x, 2x, \frac{x}{3}$ and a sphere is given to be constant. Prove that the sum of their volumes is minimum if x = 3radius of sphere. Also find the minimum value of the sum of their volumes.

62.Show that the cone of the greatest volume which can be inscribed in given sphere has an altitude equal to $\frac{2}{3}$ of the diameter of the sphere.

63.Find the shortest distance between the line $y - x = 1$ and the curve $x = y^2$.

64.Find the area of greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

65.Find the coordinates of the points on the curve $y = \frac{x}{1+x^2}$ where tangent to the curve has greatest slope.

66.A telephone company in a town has 500 subscribers on its list and collects fixed charges of Rs. 300 per subscriber per year. The company proposes to increase the annual subscription and it is believed that for every increase of Rs. 1/-, one subscriber will discontinue the service. Find what increase will bring maximum income to the company.

67.The total cost of producing x pocket radio set per day is Rs. $\left(\frac{1}{4}x^2 + 35x + 25\right)$ and the price per set at which they may be sold is Rs.$\left(50 - \frac{1}{2}x\right)$. What should be the daily output to obtain a maximum total profit?

68.A private mobile company serving a small community makes a profit of Rs.12/- per subscriber, if it has 725 subscribers. It decides to reduce the rate by a fixed amount for each subscriber over 725, there by reducing the profit by 1 paisa per subscriber. Thus

69.there will be profit of Rs.11.99 on each of 726 subscribers, Rs.11.98 on each of the 727 subscribers etc. What is the number of subscribers which will give the company the maximum profit?

70.A figure consists of a semicircle with a rectangle on its diameter. Given that the perimeter of figure is 20cm, find its dimensions in order that its area may be maximum.

71.The section of a window consists of a rectangle surmounted by an equilateral triangle. If the perimeter be given as 16m. Find the dimensions of the window in order that the maximum amount of light may be admitted.

72.A closed rectangular box with a square base is to be made so as to contain 1000 cubic feet. The cost of the material per square foot for the bottom is 15 paisa, for the top 25 paisa and for sides 20 paisa. The labour charges for making the box are Rs.3/-. Find the dimensions of the box when the cost is minimum.

73.A square tank of capacity 250 cubic m. has to dug out. The cost of land is Rs.50/- per square meter. The cost of digging increases with the depth and for the whole tank is $400(depth)^2$ rupees. Find the dimensions of the tank for the least total cost.

74.Find the dimensions of a rectangle of greatest area that can be inscribed in a semi-circle of radius r.

75.Show that a triangle of maximum area that can be inscribed in a circle of radius a is an equilateral triangle.

76.One corner of long rectangular sheet of paper of width 1 ft. is folded over so as to reach the opposite edge of the sheet. Find minimum length of the crease.

77.A conical vessel is to be prepared out of a circular sheet of gold of unit radius. How much sectorial area is to be removed from the sheet so that the vessel has maximum volume?

78.For a given curved surface of a right circular cone, show that the volume is maximum when semi-vertical angle of the cone is $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

79.A given quantity of metal is to be cast into a half cylinder i.e. with a rectangular base and semicircular ends. Show that in order that the total surface area may be minimum the ratio of the height of the cylinder to the diameter of the semicircular ends is $\frac{\pi}{\pi+2}$.

80.A cone is circumscribed to a sphere of radius r. Show that volume of the cone is minimum if its altitude is 4r and its semi-vertical angle is $\sin^{-1}\left(\frac{1}{3}\right)$.

81.If a tangent to the parabola $y^2 = 8x$ makes an angle $\frac{\pi}{4}$ with the straight line $y = 3x + 5$, then find point of contact.

82.Find the equation of normal to the curve $y = \left(\frac{1}{x}\right)^x$ at the point of its maximum.

83.Find the area of the triangle, formed by the x-axis and the tangent and the normal to the curve $y = 6x - x^2$ at the point $(5, -5)$.

84.Find the angle between two curves $y = 3^x \ and \ y = 5^x$.

Answers

1. $\frac{\pi}{3}$ 2. 1 3. $196\pi cm^2$ 4. $[0,\pi]$ 5. $\frac{1}{\pi}$ 6. $3,\frac{1}{3}$ 7. $(1,2)$

8. 89 9. $\frac{1}{e}$ 10. $2\sqrt{ab}$ 11. $\frac{1}{2}$ 12. $(-\infty,0)$ 13. 0

14. $\left(\frac{1}{2},1\right)$ 15. $a \le 0$ 16. $\frac{1}{\pi}cm/s$ 17. $\frac{3}{8\pi}cm/min$ 19. 8m/sec

20. 3000L/s 21. $10\sqrt{3}$ 22. 3km/h 23. $\frac{35}{88}m/h$ 25. $\frac{6}{49\pi}m/min$

26. $1-3x+2x^2$ 27. 72 29. 25 30. Increasing 31. Increasing for all $a \ge 1$. Decreasing for all $x \le 1$ 32. Increasing on $[0,e]$, Decreasing on $[e,0]$,

33. Increasing on $\left[0,\frac{3\pi}{4}\right] \cup \left[\frac{7\pi}{4},2\pi\right]$, Decreasing on $\left[\frac{3\pi}{4},\frac{7\pi}{4}\right]$ 36. $(-\infty,1] \cup [2,3]$

37. $[1,\infty)$ 39. Increasing on $[0,\infty)$, Decreasing $(-\infty,0]$

40. Strictly increasing $[-2,1] \cup [3,\infty)$, strictly decreasing $(-\infty,-2] \cup [1,3]$

41. $\sqrt{2}bx - ay - ab = 0$ 42. (i) $y-2x-3=0$ (ii) $36y+12x-227=0$ 43. $(4,4)$

44. $2y+x-2=0,\frac{2}{\sqrt{5}}$ 48. Decrease 72 units/sec. 51. $x+y=3, y=x-1$

52. $\left(4,\pm\frac{8}{3}\right)$ 53. 1.004343 54. $\pi\ cm^2$ 55. 0.3% 56. 0.198 57. 61. $18r^3+36+27\pi r^3$

63. $\frac{3\sqrt{2}}{8}$ 64. $2ab\ sq.units$ 65. $(0,0)$ 66. 100 67. 10 68. 963 69. $\frac{20}{\pi+4},\frac{10}{\pi+4}$

70. $\frac{16}{6-\sqrt{3}},8\left(\frac{3-\sqrt{3}}{6-\sqrt{3}}\right)$ 71. 10 feet 72. 10m 73. $\sqrt{2}r,\frac{r}{\sqrt{2}}$ 75. $\frac{3\sqrt{3}}{4}$

76. $\pi\left(1-\sqrt{\frac{2}{3}}\right)sq.units$ 80. $(0,0)\ and\ \left(\frac{1}{2},-2\right)$ 81. $x=\frac{1}{e}$ 82. $\frac{425}{8}sq.units$

83. $\tan^{-1}\left(\frac{log3-log5}{1+log3.log5}\right)$

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