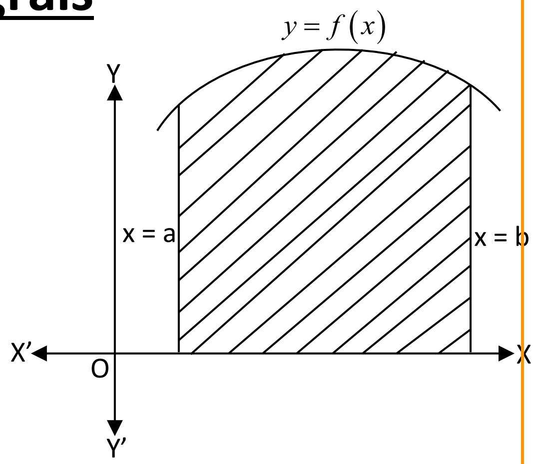
Applications Of Integrals

❖ The area bounded by the curve $y = f(x)$ above the x-axis and between the lines $x = a$ and $x = b$ is given by $\int_a^b y\,dx = \int_a^b f(x)\,dx$

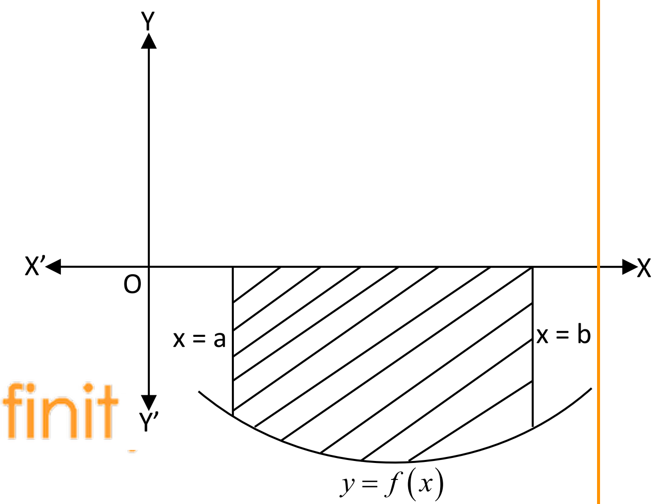


❖ If the curve between the lines $x = a$ and $x = b$ lies below the x-axis, then the required area is given by

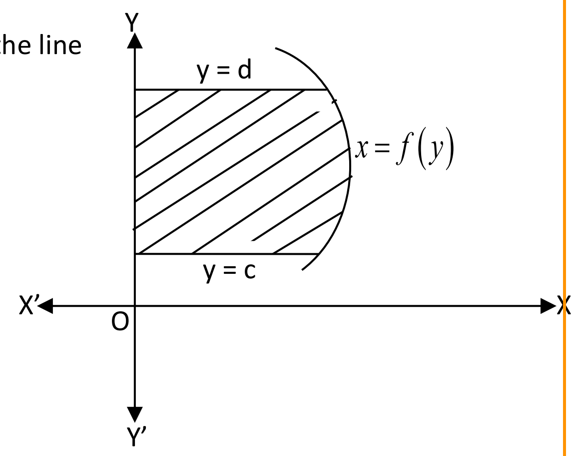
(i) $\int_a^b (-y)\,dx = -\int_a^b y\,dx = -\int_a^b f(x)\,dx$

OR

(ii) $\left|\int_a^b f(x)\,dx\right|$



❖ The area bounded by the curve $x = f(y)$ above the y-axis and the line $y = c$, $y = d$ is given by $\int_c^d x\,dy = \int_c^d f(y)\,dy$

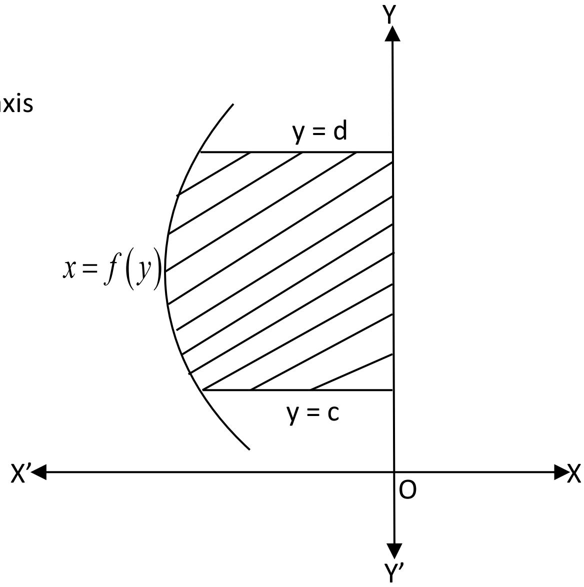


❖ If the curve between the lines $y = c$, $y = d$ lies below the y-axis (to the left of y-axis) then area is given by

(i) $$\int_c^d (-x)\,dy = -\int_c^d (x)\,dy = -\int_c^d f(y)dy$$

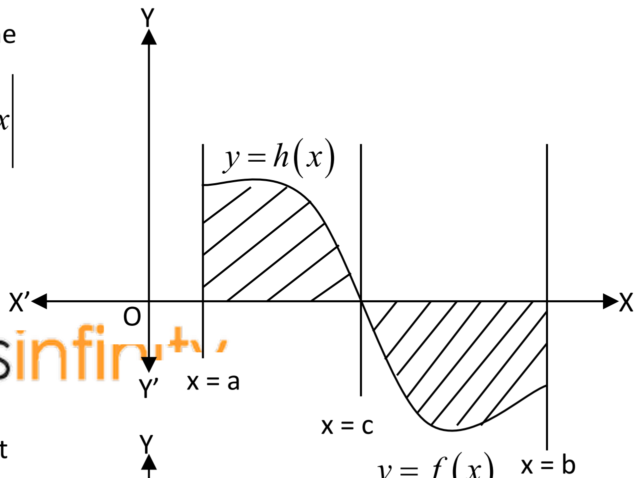
OR

(ii) $$\left|\int_c^d f(y)dy\right|$$



❖ The area bounded by the curve $y = h(x)$, x-axis and the two lines $x = a$ and $x = b$ is given by $A = \int_a^c ydx + \left|\int_c^b ydx\right|$

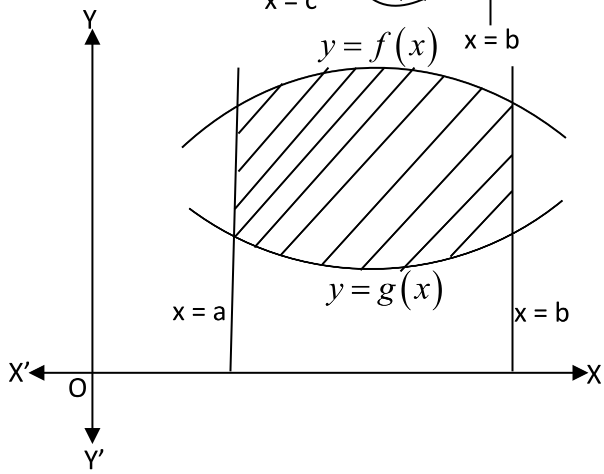
where *c* is a point in between *a* and *b*



❖ If we have two curve $y = f(x)$ and $y = g(x)$ such that $y = f(x)$ lies above the curve $y = g(x)$ and both are above the x-axis then area bounded between them and ordinates $x = a$ and $x = b$ (b > a), is given by

$$A = \int_a^b f(x)dx - \int_a^b g(x)dx$$

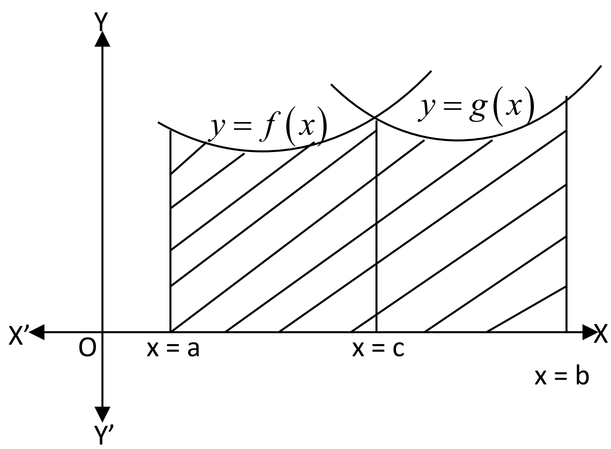
i.e. upper curve area – lower curve area.



❖ The area bounded by the curves $y = f(x)$ and $y = g(x)$ between the ordinates $x = a$ and $x = b$ is given by

$$A = \int_a^c f(x)dx + \int_c^b g(x)dx$$

where $x = c$ is the point of intersection of the two curves.



Solve the Following

1. Find the area of the region $\{(x,y): y^2 \le 4x, 4x^2+4y^2 \le 9\}$
2. Find the area of the region included between the parabola $y^2 = x$ and the line $x+y=2$.
3. Sketch the region bounded by the curve $y = 2x - x^2$ and the x-axis and its area using integration.
4. Sketch the graph $y = |x+1|$ and evaluate $\int_{-3}^{1} |x+1| dx$.
5. Using integration, find the area of the region bounded by the following lines: $y = 1+|x+1|$, $x=-3$, $x=3$ and $y=0$.
6. Using integration, find the area of the region bounded between the line $x=4$ and the parabola $y^2 = 16x$.
7. Draw a rough sketch of region $\{(x,y): y^2 \le 3x, 3x^2+3y^2 \le 16\}$ and find the area enclosed by the region using method of integration.
8. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$.
9. Draw a rough sketch of the graph of the function $y = 2\sqrt{1-x^2}$, $x \in [0,1]$ and evaluate the area enclosed between the curve and the x-axis.
10. Draw a rough sketch and find the area of the region bounded by two parabolas $y^2 = 4x$ and $x^2 = 4y$ by using method of integration.
11. Make a rough sketch of the region given below and find its area using method of integration: $\{(x,y): 0 \le y \le x^2+3, 0 \le y \le 2x+3, 0 \le x \le 3\}$.
12. Sketch the region lying in the first quadrant and bounded by $y = 9x^2$, $x=0$, $y=1$ and $y=4$. Using integration, find the area of the enclosed region.
13. Draw a rough sketch of the region enclosed between the circles $x^2+y^2=4$ and $(x-2)^2+y^2=4$. Using integration, find the area of the enclosed region.
14. Sketch a rough graph of the parabola $y = \frac{3}{4}x^2$ and the line $3x-2y+12=0$ and find area bounded by two curves.
15. Using integration, find the area of the circle $x^2+y^2=16$ which is exterior to the parabola $y^2=6x$.
16. Using integration, find the area of the region bounded by the triangle whose vertices are (-1, 1), (0, 5) and (3, 2).
17. Using integration compute the area bounded by the lines $x+2y=2$, $y-x=1$ and $2x+y=7$.
18. Find the area bounded by the curve $y=x$, $y=x^3$.
19. Find the area of the region bounded by the curves $y=x^2+2$, $y=x$, $x=0$ and $x=3$.
20. Find the area bounded by the semi-circle $y=\sqrt{4-x^2}$ and its diameter $y=0$.
21. Find the area bounded by the curve $|x|+y=1$ and axis of x.
22. Find the area bounded by the parabolas $y^2=5x+6$ and $x^2=y$.
23. Find the area bounded by the parabolas $5x^2-y=0$ and $2x^2-y+9=0$.
24. Find the ratio in which the x-axis divides the area of the region bounded by the parabolas $y=4x-x^2$ and $y=x^2-x$.
25. Find the area of the region bounded by the curves $y=|x-2|$ and $y=4-|x|$.
26. Find the area enclosed by the curves $x^2=y$, $y=x+2$ and x-axis .
27. Using integration find area of the region $\{(x,y): 0 \le y \le x^2, 0 \le y \le x+2; 0 \le x \le 3\}$
28. Using integration find area of the region $\{(x,y): |x-1| \le y \le \sqrt{5-x^2}\}$

29. Find the area of the smaller region bounded by the curves $x^2 + y^2 = 4$ and $y^2 = 3(2x-1)$.

30. Sketch the graph of $f(x) = \begin{cases} |x-2|+2, & x \le 2 \\ x^2 - 2, & x > 2 \end{cases}$ Evaluate $\int_0^4 f(x)dx$. What does the value of this integral represent on the graph?

Answers

1. $\left[\frac{\sqrt{2}}{6} + \frac{9}{4}\cos^{-1}\left(\frac{1}{3}\right)\right] sq\, units$ **2.** $\frac{7}{6} sq\, units$ **3.** $\frac{4}{3} sq\, units$ **4.** 4 **5.** 16 sq units

6. $\frac{128}{3} sq\, units$ **7.** $\left[\frac{4}{\sqrt{3}} a^{3/2} + \frac{8\pi}{3} - a\sqrt{\frac{16}{3} - a^2} - \frac{16}{3}\sin^{-1}\left(\frac{\sqrt{3}a}{4}\right)\right] sq\, units$, where $a = \frac{-9+\sqrt{273}}{6}$

8. $\left(\frac{\pi ab}{4} - \frac{1}{2}ab\right) sq\, units$ **9.** $\frac{\pi}{2} sq\, units$ **10.** $\frac{16}{3} sq\, units$ **11.** $\frac{8}{3} sq\, units$

12. $\frac{14}{9} sq\, units$ **13.** $2\left(\sqrt{3} + \frac{2\pi}{3}\right) sq\, units$ **14.** 27 sq units **15.** $\frac{4}{3}\left(8\pi - \sqrt{3}\right) sq\, units$

16. $\frac{15}{2} sq\, units$ **17.** 6 sq units **18.** $\frac{1}{2} sq\, units$ **19.** $\frac{21}{2} sq\, units$ **20.** $2\pi sq\, units$ **21.** 1 sq units

22. $\frac{27}{5} sq\, units$ **23.** $12\sqrt{3} sq\, units$ **24.** 4:129 **25.** 6 sq units **26.** $\frac{5}{6} sq\, units$

28. $\left(\frac{5\pi}{4} - \frac{1}{2}\right) sq\, units$ **30.** $\frac{62}{3} sq\, units$

By **Arun Kumar Shukla**