Continuity

4Marks

1. Show that the function $f(x) = \begin{cases} x\sin\dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at x = 0.

2. Show that the function $f(x) = \begin{cases} \dfrac{x\sin x}{x} + \cos x, & x \neq 0 \\ 2, & x = 0 \end{cases}$ is continuous at $x = 0$.

3. Show that the function $f(x) = \begin{cases} \dfrac{e^{1/x} - 1}{e^{1/x} + 1}, & when\ x \neq 0 \\ 0, & when\ x = 0 \end{cases}$ is discontinuous at $x = 0$

4. Find the value of the constant k so that the function given below is continuous at $x = 0$:

   $f(x) = \begin{cases} \dfrac{1 - \cos 2x}{2x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$ 　　　　　　(Ans. 1)

5. Find the value of a if the function f(x) defined by $f(x) = \begin{cases} 2x-1, & x < 2 \\ a, & x = 2 \\ x+1, & x > 2 \end{cases}$ is continuous

   at $x = 2$.

6. If the function f(x) defined by $f(x) = \begin{cases} \dfrac{\log(1+ax) - \log(1-bx)}{x}, & if\ x \neq 0 \\ k, & if\ x = 0 \end{cases}$ is continuous at

   $x = 0$, find k.　　　　　(Ans. k = a+b)

7. Find the values of a so that the function f(x) defined by $f(x) = \begin{cases} \dfrac{\sin^2 ax}{x^2}, & if\ x \neq 0 \\ 1, & if\ x = 0 \end{cases}$ may

   be continuous at $x = 0$.　　　　　　　　　　　(Ans. ± 1)

8. Let $f(x) = \begin{cases} \dfrac{1 - \cos 4x}{x^2}, & if\ x < 0 \\ a, & if\ x = 0 \\ \dfrac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4}, & if\ x > 0 \end{cases}$ Determine the value of a so that f(x) is continuous at

   $x = 0$.　　　　　　　　　　　　　　　　(Ans.8)

9. Determine f(0) so that the function f(x) defined by $f(x) = \dfrac{\left(4^x - 1\right)^3}{\sin\left(\dfrac{x}{4}\right)\log\left(1 + \dfrac{x^2}{3}\right)}$

   becomes continuous at $x = 0$.   (Ans. $12\left(\log_e 4\right)^3$).

10. Let $f(x) = \begin{cases} \dfrac{1 - \cos x}{x^2}, & when\ x \neq 0 \\ 1 & , when\ x = 0 \end{cases}$ Show that f (x) is discontinuous at $x = 0$.

11. Discuss the continuity of $f(x) = \begin{cases} \dfrac{e^x - 1}{\log(1 + 2x)}, & if\ x \neq 0 \\ 7 & , if\ x = 0 \end{cases}$ at $x = 0$.

12. Discuss the continuity of $f(x) = \begin{cases} \dfrac{\left|x^2 - 1\right|}{x - 1}, & for\ x \neq 1 \\ 2 & , for\ x = 1 \end{cases}$ at $x = 1$.

13. Show that $f(x) = \begin{cases} \dfrac{\sin 3x}{\tan 2x} & , if\ x < 0 \\ \dfrac{3}{2} & , if\ x = 0 \\ \dfrac{\log(1 + 3x)}{e^{2x} - 1} & , if\ x > 0 \end{cases}$ is continuous at $x = 0$.

14. Show that $f(x) = 2x + |x|$ is continuous at $x = 0$.

15. Determine the value of the constant k so that the function $f(x) = \begin{cases} \dfrac{\sin 2x}{5x}, & if\ x \neq 0 \\ k & , if\ x = 0 \end{cases}$ is

   continuous at $x = 0$.   (Ans. $\dfrac{2}{5}$)

16. Prove that the function $f(x) = \begin{cases} \dfrac{x}{|x| + 2x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$ remains discontinuous at $x = 0$,

   regardless the choice of k.

17. Determine the values of a, b, c for which the function

   $f(x) = \begin{cases} \dfrac{\sin(a + 1)x + \sin x}{x}, & for\ x < 0 \\ c & , for\ x = 0 \\ \dfrac{\sqrt{x + bx^2} - \sqrt{x}}{bx^{3/2}} & , for\ x > 0 \end{cases}$ is continuous at $x = 0$.

   $\left(Ans.\ a = -\dfrac{3}{2}, b \in R - \{0\}, c = \dfrac{1}{2}\right)$

18. If $f(x) = \begin{cases} \dfrac{1-\cos kx}{x\sin x}, & x \neq 0 \\ \dfrac{1}{2}, & x = 0 \end{cases}$ is continuous at $x = 0$, find k. $\qquad (Ans. \pm 1)$

19. If $f(x) = \begin{cases} \dfrac{x-4}{|x-4|} + a, & if\ x < 4 \\ a+b, & if\ x = 4 \\ \dfrac{x-4}{|x-4|} + b, & if\ x > 4 \end{cases}$ is continuous at $x = 4$, find a, b. $\qquad (Ans.\ a = 1, b = -1)$

20. Let $f(x) = \dfrac{\log\left(1+\dfrac{x}{a}\right) - \log\left(1-\dfrac{x}{b}\right)}{x}, x \neq 0$. Find the value of f at $x = 0$ so that f

becomes continuous at $x = 0$. $\qquad \left(Ans.\ \dfrac{a+b}{ab}\right)$

21. If $f(x) = \begin{cases} \dfrac{2^{x+2} - 16}{4^x - 16}, & if\ x \neq 2 \\ k, & if\ x = 2 \end{cases}$ is continuous at $x = 2$, find k. $\qquad \left(Ans.\ \dfrac{1}{2}\right)$

22. If $f(x) = \begin{cases} \dfrac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2+1} - 1}, & if\ x \neq 0 \\ k, & if\ x = 0 \end{cases}$ is continuous at $x = 0$, find k. $\quad (Ans. -4)$

23. For what value of k, is the following function continuous at $x = 0$:

$$f(x) = \begin{cases} \dfrac{1-\cos 4x}{8x^2}, & if\ x \neq 0 \\ k, & if\ x = 0 \end{cases}$$

24. Find the point of discontinuity if any of the function $f(x) = \begin{cases} \dfrac{e^x - 1}{\log_e (1+2x)}, & if\ x \neq 0 \\ 7, & if\ x = 0 \end{cases}$

25. Find the value of p if $f(x) = \begin{cases} \dfrac{\sqrt{1+px} - \sqrt{1-px}}{x}, & if\ -1 \leq x < 0 \\ \dfrac{2x+1}{x-2}, & if\ 0 \leq x \leq 1 \end{cases}$ is continuous.

$\left(Ans.\ p = -\dfrac{1}{2}\right)$

26. If $f(x) = \begin{cases} \dfrac{x^2}{a}, & if\ 0 \leq x < 1 \\ a, & if\ 1 \leq x < \sqrt{2} \\ \dfrac{2b^2 - 4b}{x^2}, & if\ \sqrt{2} \leq x < \infty \end{cases}$ is continuous on $[0, \infty)$, then find the most suitable

values of a and b. $\qquad \left(Ans.\ a = -1, b = 1\ or\ a = 1, b = 1 \pm \sqrt{2}\right)$

27. If the function $f(x) = \begin{cases} 3ax+b & , if\ x > 1 \\ 11 & , if\ x = 1 \\ 5ax - 2b & , if\ x < 1 \end{cases}$ is continuous at $x = 1$, find the value of a and

b. $\left(Ans.\ a = 3, b = 2 \right)$

28. Find all the points of discontinuity of the function $f(x) = \left[x^2 \right]$ on $[1, 2)$, where $\left[\ \cdot\ \right]$ the

greatest integer function. $\left(Ans.\ \sqrt{2}, \sqrt{3} \right)$

29. Discuss the continuity of $f(x) = \begin{cases} \dfrac{1-x^n}{1-x} & , x \neq 1 \\ n-1 & , x = 1 \end{cases}$ $n \in N$ at $x = 1$

30. Discuss the continuity of $f(x) = \begin{cases} |x| \cos\left(\dfrac{1}{x}\right) & , x \neq 0 \\ 0 & , x = 0 \end{cases}$ at $x = 0$

Prepared by **Arun Kumar Shukla**