Definite Integrals

The Definite Integral

Let $F(x)$ be any antiderivative of $f(x)$, then for any two values of the independent variable x, say *a* and *b* the difference $F(b)-F(a)$ is called the **definite integral** of $f(x)$ from *a* to *b* and is denoted by $\int_a^b f(x)dx$. Thus

$$\int_a^b f(x)dx = F(b)-F(a)$$

Where $F(x)$ is any antiderivative of $f(x)$. The numbers *a* and *b* are called the **limits of integration**; *a* is the lower limit and *b* is the upper limit.

Properties of Definite Integrals

1. $\int_a^b f(x)dx = -\int_b^a f(x)dx$
2. $\int_a^b f(x)dx = \int_a^b f(t)dt$
3. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, \text{where } a < c < b$
4. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$
5. $\int_0^a f(x)dx = \int_0^a f(a-x)dx$
6. $$\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)\,dx & \text{if } f(-x)=f(x) \text{ i.e. } f(x) \text{ is even} \\ 0 & \text{if } f(-x)=-f(x) \text{ i.e. } f(x) \text{ is odd} \end{cases}$$
7. $$\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)\,dx & \text{if } f(2a-x)=f(x) \\ 0 & \text{if } f(2a-x)=-f(x) \end{cases}$$

Removal of x: Let $I=\int_0^a f(x)dx$ where $f(x)$ is function of x whose integral is known and $f(a-x)=f(x)$. Then

$$I=\int_0^a (a-x)f(a-x)dx$$
$$=\int_0^a (a-x)f(x)dx$$

$$= a\int_0^a f(x)dx - \int_0^a xf(x)dx$$

$$I = a\int_0^a f(x)dx - I$$

$$\therefore 2I = a\int_0^a f(x)dx$$

$\Rightarrow I = \frac{a}{2}\int_0^a f(x)dx$, Now $\int_0^a f(x)dx$ can be evaluated as $f(x)$ is known as integrable function.

1/2 Marks

1. $\int\left(\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}\right)dx$
2. $\int_{-1}^{1} e^{|x|}dx$
3. $\int\frac{1}{1-\sin^2 x}dx$
4. $\int\left(8^x + x^8 + \frac{8}{x} + \frac{x}{8}\right)dx$
5. $\int_{-1}^{1}\left(x^{99}\cos^4 x\right)dx$
6. $\int\frac{1}{x\log x\log(\log x)}dx$
7. $\int_0^{\frac{\pi}{2}}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$
8. $\int\left(e^{a\log x} + e^{x\log a}\right)dx$
9. $\int\left(\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}\right)dx$
10. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^7 x\,dx$
11. $\int\left(x^c + c^x\right)dx$
12. $\frac{d}{dx}\left[\int f(x)dx\right]$
13. $\int\frac{1}{\sin^2 x\cos^2 x}dx$
14. $\int e^{-\log e^x}dx$
15. $\int\frac{e^x}{a^x}dx$
16. $\int 2^x e^x dx$
17. $\int\frac{e^{\sqrt{x}}}{\sqrt{x}}dx$
18. $\int\cos^2\alpha\,dx$
19. $\int\frac{1}{x\cos\alpha + 1}dx$
20. $\int\sec x\log(\sec x + \tan x)dx$
21. $\int\frac{1}{\cos\alpha + x\sin\alpha}dx$
22. $\int\left(\frac{x+1}{x}\right)(x + \log x)\,dx$
23. $\int\left(\sqrt{ax} - \frac{1}{\sqrt{ax}}\right)^2 dx$
24. $\int_0^{\pi}|\cos x|dx$
25. $\int_0^2[x]dx$, where [ ] is greatest integer function
26. $\int_0^{\sqrt{2}}\left[x^2\right]dx$, where [ ] is greatest integer function
27. $\int_a^b\frac{f(x)}{f(x) + f(a+b-x)}dx$

28. $\int_{-2}^{1} \frac{|x|}{x} dx$

29. $\int_{-1}^{1} x|x|dx$

30. If $\int_{0}^{a} \frac{1}{1+x^2} dx = \frac{\pi}{4}$ then find the value of a.

31. Evaluate: $\int_{a}^{b} f(x)dx + \int_{b}^{a} f(x)dx$

32. $\int_{-1}^{3} \left[\tan^{-1}\left(\frac{x}{x^2+1}\right) + \tan^{-1}\left(\frac{x^2+1}{x}\right) \right] dx$

33. $\int_{-\frac{1}{2}}^{\frac{1}{2}} (\cos x)\left[\log\left(\frac{1-x}{1+x}\right)\right] dx$

34. $\int_{-\pi}^{\pi} (1-x^2)\sin x \cos^2 x \, dx$

35. $\int_{-2}^{1} \frac{|x|}{x} dx$

36. $\int_{0}^{2} [x^2] dx$

37. $\int_{0}^{2\pi} (\sin x + |\sin x|) dx$

38. $\int_{-2}^{2} |[x]| dx$

39. $\int \frac{d(\sin x)}{\sqrt{1-\sin^2 x}}$

40. $\int \sqrt{1+\sin\frac{x}{4}}\, dx$

41. $\int \frac{a^{\frac{x}{2}}}{\sqrt{a^{-x}-a^{x}}} dx$

42. $\int e^{x\log a} e^{x} dx$

43. $\int \cos^{-3/7} x \sin^{-11/7} x \, dx$

44. $\int \frac{dx}{\sqrt[4]{(x+1)^5 (x+2)^3}}$

45. $\int \frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}} dx$

46. $\int \sec^{8/9} x \cos ec^{10/9} x \, dx$

47. $\int_{0}^{1} [2x] dx$, where [ ] is greatest integer function

48. $\int e^{\log(x+1)-\log x} dx$

49. $\int e^{\log x + \log\sin x} dx$

50. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} |\sin x| dx$

4/6 Marks

(A) Evaluate each of the following definite integrals as limit of sum:

1. $\int_{a}^{b} x \, dx$

2. $\int_{1}^{2} (x+5) dx$

3. $\int_{0}^{2} (x^2+1) dx$

4. $\int_{1}^{3} (2x^2+5x) dx$

5. $\int_{1}^{2} x^3 \, dx$

6. $\int_{0}^{1} (3x^2+2x+1) dx$

7. $\int_{1}^{3} (x^2-x+5) dx$

8. $\int_{a}^{b} \frac{1}{x^2} dx$

9. $\int_{a}^{b} e^x \, dx$

10. $\int_{0}^{1} e^{2-3x} \, dx$

11. $\int_{-1}^{1} e^x\,dx$

12. $\int_{0}^{4} \left(x+e^{2x}\right)dx$

13. $\int_{2}^{4} 2^x\,dx$

(B) Evaluate each of the following definite integrals:

1. $\int_{0}^{1} \frac{2x+3}{5x^2+1}dx$

2. $\int_{1}^{2} \frac{5x^2}{x^2+4x+3}dx$

3. $\int_{2}^{4} \frac{x^3}{\sqrt{x^4-1}}dx$

4. $\int_{0}^{1} \frac{x}{1+\sqrt{x}}dx$

5. $\int_{2}^{5} \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^3 dx$

6. $\int_{0}^{1} \sqrt{\frac{1-x}{1+x}}\,dx$

7. $\int_{0}^{\frac{\pi}{2}} \sqrt{1+\sin x}\,dx$

8. $\int_{0}^{\frac{\pi}{2}} \cos^3 x\left(\sin x\right)^{\frac{1}{4}}dx$

9. $\int_{0}^{\frac{\pi}{2}} \frac{1}{a^2\sin^2 x+b^2\cos^2 x}dx$

10. $\int_{0}^{1} \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$

11. $\int_{0}^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1} x}{\left(1-x^2\right)^{3/2}}dx$

12. $\int_{\frac{\pi}{2}}^{\pi} e^x\left(\frac{1-\sin x}{1+\cos x}\right)dx$

13. $\int_{1}^{2} e^{2x}\left(\frac{1}{x}-\frac{1}{2x^2}\right)dx$

(C) Using the properties of definite integrals, evaluate each of the following:

1. $\int_{0}^{\frac{\pi}{2}} \frac{\sin x}{\sin x+\cos x}dx$

2. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\tan x}}dx$

3. $\int_{0}^{\frac{\pi}{2}} \frac{1}{1+\tan x}dx$

4. $\int_{0}^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x+\cos^5 x}dx$

5. $\int_{0}^{\frac{\pi}{2}} \frac{\sin x-\cos x}{1+\sin x.\cos x}dx$

6. $\int_{0}^{\pi} \cos^5 x\,dx$

7. $\int_{0}^{\pi} \frac{x}{a^2\cos^2 x+b^2\sin^2 x}dx$

8. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x\,dx$

9. $\int_{0}^{\pi} \frac{x}{1+\sin x}dx$

10. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\cot^{3/2} x}dx$

11. $\int_0^{\frac{\pi}{2}} \frac{\tan^3 x}{1+\tan^3 x}\,dx$

12. $\int_0^{\frac{\pi}{2}} \log \tan x\,dx$

13. $\int_0^{2\pi} \sin^7 x\,dx$

14. $\int_{-1}^{1} \left(\frac{2-x}{2+x}\right) dx$

15. $\int_0^1 \log\left(\frac{1}{x}-1\right) dx$

16. $\int_0^2 x\sqrt{2-x}\,dx$

17. $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x+\cos x}\,dx$

18. $\int_0^1 \frac{\log(1+x)}{x^2}\,dx$

19. $\int_0^{\frac{\pi}{2}} \frac{x}{\sin x+\cos x}\,dx$

20. $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\,dx$

21. $\int_0^{\pi} \frac{x}{a^2-\cos^2 x}\,dx$

22. $\int_{-1}^{1} \frac{|x|}{x}\,dx$

23. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^4 \sin^7 x\,dx$

24. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\sin x|\,dx$

25. $\int_0^{\pi} |\cos x|\,dx$

26. $\int_1^4 |x-2|\,dx$

27. $\int_{-1}^{2} |x^3-x|\,dx$

28. $\int_{-1}^{\frac{3}{2}} |x\sin \pi x|\,dx$

29. $\int_0^{\frac{\pi}{2}} |\sin x.\cos x|\,dx$

30. $\int_{1/4}^{1} |2x-1|\,dx$

31. $\int_0^1 x(1-x)^n\,dx$

32. $\int_0^{\frac{\pi}{2}} \log \sin x\,dx$

33. $\int_0^{\frac{\pi}{4}} \log(1+\tan\theta)\,d\theta$

34. $\int_0^{\frac{\pi}{2}} \log(\tan x+\cot x)\,dx$

35. $\int_0^{\pi} \log(1+\cos x)\,dx$

36. $\int_0^{\frac{\pi}{2}} \frac{x\sin x.\cos x}{\sin^4 x+\cos^4 x}\,dx$

37. $\int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{9+16\sin 2x}dx$

38. $\int_0^{\pi} \frac{x\tan x}{\sec x.\cos ecx}\,dx$

39. $\int_0^{\frac{\pi}{2}} (2\log\sin x-\log\sin 2x)\,dx$

40. $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$

41. $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x+4\sin^2 x}\,dx$

42. $\int_0^{\pi} \frac{1}{5+3\cos x}\,dx$

43. $\int_0^{\frac{\pi}{2}} \frac{x}{1+\sin x+\cos x}\,dx$

44. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x+\cos x}{\sqrt{\sin 2x}}\,dx$

45. $\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}\,dx$

46. $\int_{-1}^{2} f(x)\,dx$, when $f(x)=\begin{cases}2x+1, x\le 1\\ x-5, x>1\end{cases}$

47. $\int_1^4 f(x)dx$, when $f(x)=\begin{cases}7x+3, 1\le x\le 3\\ 8x, 3<x\le 4\end{cases}$

48. $\int_0^9 f(x)dx$, when $f(x)=\begin{cases}\sin x, 0\le x<\frac{\pi}{2}\\ 1, \quad \frac{\pi}{2}\le x\le 3\\ e^{x-3}, 3<x\le 9\end{cases}$

49. $\int_2^8 f(x)dx$, when $f(x)=\begin{cases}3x+4, 2\le x\le 3\\ x^2+4, 3<x\le 8\end{cases}$

50. $\int_1^4 \left[|x|+|x-3|\right]dx$

51. $\int_1^4 f(x)dx$, when $f(x)=|x-1|+|x-2|+|x-3|$

52. Prove that $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$

53. Prove that $\int_0^{2a} f(x)\,dx=\int_0^a f(x)+\int_0^a f(2a-x)\,dx$

54. If $f(x)=a+bx+cx^2$, show that $\int_0^1 f(x)\,dx=\frac{1}{6}\left[f(0)+f\left(\frac{1}{2}\right)+f(1)\right]$

55. If $\int_0^a 3x^2dx=8$, find the value of a

56. If $\int_a^b x^3dx=0$ and $\int_a^b x^2dx=\frac{2}{3}$, find the value of a and b.

57. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[\sin|x|-\cos|x|\right]dx$

58. $\int_0^1 \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$

59. $\int_0^{\frac{\pi}{2}} \sin 2x \log\tan x\,dx$

60. $\int_0^{\pi} \frac{x\tan x}{\sec x+\cos ecx}\,dx$

61. $\int_0^{\pi} \left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx$

62. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \log\sin x\,dx$

63. $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}\,dx$

64. $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x+\cos x}\,dx$

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65. $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

66. $\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\cos^4 x + \sin^4 x} dx$

67. $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$

68. $\int_{-1}^{1} \log\left(\frac{1+\sin x}{1-\sin x}\right) dx$

69. $\int_0^{1} \cot^{-1}\left(1-x+x^2\right) dx$

70. $\int_0^{1} \frac{\log(1+x)}{1+x^2} dx$

71. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{\frac{7}{2}}} dx$

72. $\int_0^{\frac{3}{2}} |x \cos \pi x| dx$

73. $\int_0^{2} |x^2 + 2x - 3| dx$

74. $\int_3^{9} \frac{\sqrt{12-x}}{\sqrt{x}+\sqrt{12-x}} dx$

75. $\int_0^{\pi} |\sin^3 \theta| d\theta$

76. If $f(x) = \begin{cases} |x|, -1 \le x \le 1 \\ |x-2| \end{cases}$, then Evaluate $\int_{-1}^{3} f(x) dx$

77. $\int_0^{1} \frac{x}{\left[x+\sqrt{1-x^2}\right]\sqrt{1-x^2}} dx$

78. $\int_0^{\frac{\pi}{2}} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}} dx$

79. $\int_0^{\frac{\pi}{8}} \cos^3 4\theta d\theta$

80. $\int_0^{1} \tan^{-1}\left(\frac{1}{x^2 - x + 1}\right) dx$

81. Prove that $\int_0^{a} f(x) dx = \int_0^{a} f(a-x) dx$ and hence prove that $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx = \frac{\pi}{4}$

82. Prove that if f is an odd function, then $\int_{-a}^{a} f(x) dx = 0$ use it to evaluate $\int_{-1}^{1} \log\left(\frac{2+x}{2-x}\right) dx$

ANSWERS

1 Mark

1. $\frac{\pi}{2}x + c$
2. $2e - 2$
3. $\tan x + c$
4. $\frac{8^x}{\log 8} + \frac{x^9}{9} + 8\log|x| + \frac{x^2}{16} + c$
5. 0
6. $\log|\log(\log x)| + c$
7. 0
8. $\frac{x^{a+1}}{a+1} + \frac{a^x}{\log a} + c$
9. $\tan x + c$
10. 0
11. $\frac{x^{c+1}}{c+1} + \frac{c^x}{\log c} + C$
12. $f(x) + c$
13. $\tan x - \cot x + c$
14. $\log|x| + c$
15. $\frac{\left(\frac{e}{a}\right)^x}{\log\left(\frac{e}{a}\right)} + c$
16. $\frac{2^x e^x}{\log(2e)} + c$
17. $2e^{\sqrt{x}} + c$
18. $x\cos^2 \alpha + c$

19. $\frac{\log|x\cos\alpha+1|}{\cos\alpha}+c$

20. $\frac{\left(\log|\sec x+\tan x|\right)^2}{2}+c$

21. $\frac{\log|\cos\alpha+x\sin\alpha|}{\sin\alpha}+c$

22. $\frac{(x+\log x)^2}{2}+c$

23. $\frac{ax^2}{2}+\frac{\log|ax|}{a}-2x+c$

24. 0

25. 1

26. $\left(\sqrt{2}-1\right)$

27. $\frac{b-a}{2}$

28. -1

29. 0

30. 1

31. 0

32. 2π

33. 0

34. 0

35. -1

36. $-\sqrt{2}-\sqrt{3}+5$

37. 4

38. 4

39. $x+c$

40. $a\left(\sin\frac{x}{8}-\cos\frac{x}{8}\right)+c$

41. $\frac{1}{\log a}\sin^{-1}\left(a^x\right)+c$

42. $\frac{(ae)^x}{\log_e ae}+c$

43. $-\frac{7}{4}\tan^{-\frac{4}{7}}x+c$

44. $-4\left(\frac{x+2}{x+1}\right)^{\frac{1}{4}}+c$

45. $\frac{2}{3}(1+x)^{\frac{3}{2}}+c$

46. $-9(\cot x)^{\frac{1}{9}}+c$

47. $\frac{1}{2}$

48. $x+\log x+c$

49. $-x\cos x+\sin x+c$

50. $2-\sqrt{2}$

Section-(A)

1. $\frac{b^2-a^2}{2}$

2. $\frac{13}{2}$

3. $\frac{14}{3}$

4. $\frac{112}{3}$

5. $\frac{15}{4}$

6. 3

7. $\frac{44}{3}$

8. $\frac{1}{a}-\frac{1}{b}$

9. e^b-e^a

10. $\frac{1}{3}\left(e^2-\frac{1}{e}\right)$

11. $\frac{e^2-1}{e}$

12. $\frac{15+e^8}{2}$

13. $\frac{12}{\log 2}$

Section-(B)

1. $\frac{1}{5}\log 6+\frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}$

2. $5-\frac{5}{2}\left(9\log\frac{5}{4}-\log\frac{3}{2}\right)$

3. $\frac{1}{2}\left(\sqrt{255}-\sqrt{15}\right)$

4. $\frac{5}{3}-2\log 2$

5. $\frac{1}{5}\left[128\sqrt{5}-53\sqrt{2}\right]$

6. $\left(\frac{\pi}{2}-1\right)$

7. 2

8. $\frac{34}{55}$

9. $\frac{\pi}{2ab}$

10. $\left(\frac{\pi}{2}-\log 2\right)$

11. $\left(\frac{\pi}{4}-\frac{1}{2}\log 2\right)$

12. $e^{\frac{\pi}{2}}$

13. $\frac{e^2\left(e^2-2\right)}{4}$

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Section-(C)

1. $\frac{\pi}{4}$ 2. $\frac{\pi}{12}$ 3. $\frac{\pi}{4}$ 4. $\frac{\pi}{4}$

5. 0 6. 0 7. $\frac{\pi^2}{2ab}$ 8. $\frac{\pi}{2}$

9. π 10. $\frac{\pi}{12}$ 11. $\frac{\pi}{4}$ 12. 0

13. 0 14. 0 15. 0 16. $\frac{16\sqrt{2}}{15}$

17. $\frac{1}{\sqrt{2}}\log\left(\sqrt{2}+1\right)$ 18. $\frac{\pi}{8}\log 2$ 19. $\frac{\pi}{2\sqrt{2}}\log\left(1+\sqrt{2}\right)$ 20. $\frac{9}{\sqrt{2}}$

21. $\frac{\pi^2}{2a\sqrt{a^2-1}}$ 22. 0 23. 0 24. 2

25. 2 26. $\frac{5}{2}$ 27. $\frac{11}{4}$ 28. $\frac{3\pi+1}{\pi^2}$

29. $\frac{1}{2}$ 30. $\frac{5}{16}$ 31. $\frac{1}{(n+1)(n+2)}$ 32. $-\frac{\pi}{2}\log 2$

33. $\frac{\pi}{8}\log 2$ 34. $\pi\log 2$ 35. $-\pi\log 2$ 36. $\frac{\pi^2}{16}$

37. $\frac{1}{40}\log 9$ 38. $\frac{\pi^2}{4}$ 39. $\frac{\pi}{2}\log 2$ 40. $\frac{\pi^2}{4}$

41. $\frac{\pi}{6}$ 42. $\frac{\pi}{4}$ 43. $\frac{\pi}{4}\log 2$

44. $2\sin^{-1}\left\{\frac{1}{2}\left(\sqrt{3}-1\right)\right\}$ 45. $\frac{\pi}{2}(\pi-2)$ 46. $-\frac{3}{2}$

47. 62 48. $3+e^6-\frac{\pi}{2}$ 49. $\frac{1159}{2}$ 50. 10

51. $\frac{19}{6}$ 55. 2 56. $a=-1, b=1$ 57. $\frac{\pi^2}{4}$

58. $\pi\sqrt{2}$ 59. $\frac{1}{4}\log 2-\frac{\pi}{8}+\frac{1}{4}$ 60. $a\pi$ 61. $\frac{1}{\sqrt{2}}\log\left(\sqrt{2}+1\right)$

62. $\frac{\pi^2}{4}$ 63. $\frac{\pi}{2}$ 64. $-\frac{\pi}{4}$ 65. $\frac{\pi^2}{4}$

66. $\frac{\pi}{2}$ 67. $\frac{\pi}{2}$ 68. 0 69. $\frac{\pi}{2}-\log 2$

70. $\frac{\pi}{8}\log 2$ 71. $\frac{7}{3}$ 72. $\frac{5\pi-2}{2\pi^2}$ 73. 4

74. 3 75. $\frac{4}{3}$ 76. 2 77. $\frac{\pi}{4}$

78. $\frac{\pi}{4}$ 79. $\frac{1}{6}$ 80. $\frac{\pi}{2}-\log 2$


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