Differential Equation

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1/2Marks

1. What is the degree of the following differential equation?
$$5x\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} - 6y = \log x$$
2. Write the order and degree of the differential equation
$$\left(\frac{d^2y}{dx^2}\right)^3 - 5\frac{dy}{dx} + 6 = 0$$
3. Write the order and degree of the differential equation
$$x - \cos\left(\frac{dy}{dx}\right) = 0$$
4. Write the order and degree of the differential equation
$\frac{dy}{dx} + \sin\left(\frac{dy}{dx}\right) = 0.$
5. Find the order and degree of the differential equation
$y = px + \sqrt{1+p^2}$ where $p = \frac{dy}{dx}$

4 Marks

Solve the following differential equations:

6. $(1+e^{2x})\,dy + (1+y^2)e^x\,dx = 0$, given that when $x = 0, y = 1$.
7. $x\frac{dy}{dx} = y - \sqrt{x^2+y^2}$
8. $\frac{dy}{dx} + y\cot x = 2x + x^2\cot x$, given that $y(0) = 0$
9. $(x^2+xy)\,dy = (x^2+y^2)\,dx$
10. $\frac{dy}{dx} + \frac{4x}{x^2+1}y + \frac{1}{(x^2+1)^2} = 0$
11. $2x^2\frac{dy}{dx} - 2xy + y^2 = 0; y(e) = e$
12. $2xy\,dx + (x^2+2y^2)\,dy = 0$
13. $\sin x\,\frac{dy}{dx} + (\cos x)y = \cos x.\sin^2 x$
14. $(y^2-x^2)\,dy = 3xy\,dx$
15. $\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$
16. $3e^x\tan y\,dx + (1-e^x)\sec^2 y\,dy = 0$, given that $y = \frac{\pi}{4}$, when $x = 1$
17. $\frac{dy}{dx} = e^{y+x} + e^y x^2$
18. $\frac{dy}{dx} = \frac{x.e^x\log x + e^x}{x\cos y}$
19. $\cos x\,\frac{dy}{dx} + y = \sin x$, given that $y = 2,\ when\ x = 0.$
20. $(x^2-1)\frac{dy}{dx} + 2xy = \frac{2}{x^2-1}$
21. $x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$
22. $\frac{dy}{dx} + y = \cos x - \sin x$
23. $\frac{dy}{dx} - \frac{y}{x} + \text{cosec}\left(\frac{y}{x}\right) = 0;$
$y = 0\ when\ x = 1$
24. $x\log x\,\frac{dy}{dx} + y = \frac{2}{x}\log x$
25. $\frac{dy}{dx} + y\cot x = 4x\,\text{cosec}\,x, (x \neq 0)$, given that $y = 0, when\ x = \frac{\pi}{2}$
26. $xy\frac{dy}{dx} = (x+2)(y+2)$, find the solution curve passing through the point $(1,-1)$.
27. $\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0.$
28. $(x^3+x^2+x+1)\frac{dy}{dx} = 2x^2 + x$
29. $e^x\tan y\,dx + (1-e^x)\sec^2 y\ \ dy = 0$
30. $(y+3x^2)\frac{dx}{dy} = x$
31. $(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$, given $y = 0,\ when\ x = 1.$
32. $\frac{dy}{dx} + 2y\tan x = \sin x$, given that $y = 0,\ when\ x = \frac{\pi}{3}.$
33. $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that $y = 1,\ when\ x = 0.$
34. $(x+1)\frac{dy}{dx} = 2e^{-y} - 1;$
$y = 0\ when\ x = 0.$
35. Find the particular solution of the differential equation:
$x(x^2-1)\frac{dy}{dx} = 1;$
$y = 0\ when\ x = 2$
36. Find the particular solution of the following differential equation:
$e^x\sqrt{1-y^2}\,dx + \frac{y}{x}dy = 0,$
$x = 0, y = 1$
37. $\frac{dy}{dx} = \sin^3 x\,.\cos^3 x + x.e^x$
38. $\left(1+e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy = 0.$

39. $\frac{dy}{dx} = (3x + y + 4)^2$
40. $2ye^{\frac{x}{y}}\,dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0.$
41. $(x - y^3)\,dy + y\,dx = 0.$
42. $xe^{\frac{y}{x}} - y + xy' = 0, y(e) = 0.$
43. $\sqrt{1 - y^2}\,dx = (\sin^{-1} y - x)\,dy,$ $y(0) = 0$
44. $x\frac{dy}{dx} + \frac{y}{\log x} = 1, given\ that\ y(1) = 1.$
45. $\sec^2 y(1 + x^2)\,dy + 2x \tan y\,dx = 0,$ given that $y = \frac{\pi}{4}$, when $x = 1.$
46. $\frac{dy}{dx} = \frac{y}{x} + \log x.$
47. $y^2 + x^2\frac{dy}{dx} = xy\frac{dy}{dx}$, given that when $x = 1,\ y = 1.$
48. $(1 + y^2)(1 + \log x)\,dx + x\,dy = 0$,given that when $x = 1, y = 1.$
49. $\frac{dy}{dx} - 3y \cot x = \sin 2x$, given that $y = 2,\ when\ x = \frac{\pi}{2}.$
50. $\frac{dy}{dx} = \frac{e^x(\sin^2 x + \sin 2x)}{y(2\log y + 1)}$
51. $(1 + \sin^2 x)\,dy + (1 + y^2)\cos x\,dx = 0,$ given that when $x = \frac{\pi}{2}$, $y = 0.$
52. $\frac{dy}{dx} = x - 1 + xy - y.$
53. $x\frac{dy}{dx} = y(\log y - \log x + 1)$
54. $y + x\,.\sin\left(\frac{y}{x}\right) = x\frac{dy}{dx}$
55. $\log\left(\frac{dy}{dx}\right) = ax + by.$
56. $\frac{dy}{dx} + 1 = e^{x+y}.$
57. $\frac{dy}{dx} = \tan(x + y).$
58. $\frac{dy}{dx} = \cos^3 x\,.\sin^4 x + x\sqrt{2x - 1}\,.$
59. $(y + x)\frac{dy}{dx} = y - x\,.$
60. $y - x\frac{dy}{dx} = x + y\frac{dy}{dx}.$
61. Form the differential equation representing the family of curves $y^2 - 2ay + x^2 = a^2$, where a is an arbitrary constant.
62. Find the differential equation for the family of circles which pass through the origin and have their centres on the x-axis.
63. Form the differential equation of the family of circles touching the y-axis at origin.
64. Show that differential equation: $(x - y)\frac{dy}{dx} = x + 2y$ is homogeneous and solve it.
65. Show that the following differential equation is homogenous, and then solve it:
$$y\,dx + x\log\left(\frac{y}{x}\right)dy - 2x\,dy = 0.$$
66. Form the differential equation of the family of parabolas having vertex at the origin and axis along positive y-axis.

Answers

1. 1
2. $Order = 2, Degree = 3$
3. $Order = 1, Degree = 1$
4. $Order = 1, Degree = Not\ defined$
5. $Order = 1, Degree = 2$
6. $\tan^{-1} y + \tan^{-1} e^x = \frac{\pi}{2}$
7. $y + \sqrt{x^2 + y^2} = C$
8. $y = x^2$
9. $\log|x| - 2\log|x - y| - \frac{y}{x} = C$
10. $(x^2 + 1)^2 y = -x + C$
11. $2x = y(\log|x| + 1)$
12. $3x^2y + 2y^3$
13. $y = \frac{1}{3}\sin^2 x + C\ \mathrm{cosec}\, x$
14. $x^2y^2(y^2 - 4x^2)^3 = C$
15. $\tan(x + y) - \sec(x + y) = x + C$
16. $(1 - e)^3 \tan y = (1 - e^x)^3$
17. $e^x + e^{-y} + \frac{x^3}{3} = c$
18. $\sin y = e^x.\log x + c$
19. $y(\sec x + \tan x) = \sec x + \tan x - x + 1$
20. $(x^2 - 1)y = \log\left|\frac{x-1}{x+1}\right| + c$
21. $x.\sin\left(\frac{y}{x}\right) = c$
22. $y = \cos x + ce^{-x}$
23. $\cos\left(\frac{y}{x}\right) = \log|x| + 1$
24. $y\log|x| = \frac{-2\log|x|}{x} - \frac{2}{x} + c$
25. $\sin x\,.\,y = 2x^2 - \frac{\pi^2}{2}$
26. $y - 2\log|y + 2| = x + 2\log|x| - 2$
27. $\sqrt{1 + x^2} + \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right| + \sqrt{1 + y^2} = c$
28. $y = \frac{1}{2}\log|x + 1| + \frac{3}{4}\log|x^2 + 1| - \frac{1}{2}\tan^{-1} x + c$
29. $\tan y = c(1 - e^x)$
30. $y = 3x^2 + cx$
31. $(1 + x^2).y = \tan^{-1} x - \frac{\pi}{4}$
32. $y = \cos x + c\cos^2 x$
33. $\tan^{-1} y = x + \frac{x^3}{3} + \frac{\pi}{4}$
34. $(2 - e^y)(x + 1) = 2$

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35. $y = \frac{1}{2}\log\left|1 - \frac{1}{x^2}\right| - \frac{1}{2}\log\frac{3}{4}$
36. $\sqrt{1-y^2} = xe^x - e^x + 1$
37. $y = \frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + e^x(x-1) + c$
38. $x + ye^{\frac{x}{y}} = c$
39. $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{3x+y+4}{\sqrt{3}}\right) = x + c$
40. $2e^{\frac{x}{y}} + \log|y| = c$
41. $x = \frac{y^2}{4} + \frac{c}{y}$
42. $y = -x\log|\log x|$
43. $x + 1 - \sin^{-1} y = e^{-\sin^{-1} y}$
44. $y = \frac{1}{2}\log|x|$
45. $(1 + x^2)\tan y = 2$
46. $y = \frac{x}{2}(\log|x|)^2 + cx$
47. $\frac{y}{x} = \log|x| + 1$
48. $\tan^{-1} y = -\log|x| - \frac{(\log|x|)^2}{2} + \frac{\pi}{4}$
49. $y = 4\sin^3 x - 2\sin^2 x$
50. $y^2\log|y| = \sin^2 x \,.\, e^x + c$
51. $\tan^{-1}(\sin x) + \tan^{-1} y = \frac{\pi}{4}$
52. $\log|y+1| = \frac{x^2}{2} - x + c$
53. $y = x.e^{cx}$
54. $\text{cosec}\left(\frac{y}{x}\right) - \cot\left(\frac{y}{x}\right) = cx$
55. $-\frac{1}{b}e^{-by} = \frac{1}{a}e^{ax} + c$
56. $(x + c)e^{x+y} + 1 = 0$
57. $y - x + \log|\sin(x+y) - \cos(x+y)| = c$
58. $y = \frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + \frac{x}{3}(2x-1)^{\frac{3}{2}} - \frac{1}{15}(2x-1)^{\frac{5}{2}} + c$
59. $\frac{1}{2}\log|x^2 + y^2| + \tan^{-1}\frac{y}{x} = c$
60. $\frac{1}{2}\log|x^2 + y^2| + \tan^{-1}\frac{y}{x} = c$
61. $(x^2 - 2y^2)(y')^2 - 4xyy' - x^2 = 0$
62. $2xyy' - y^2 + x^2 = 0$
63. $2xyy' - y^2 + x^2 = 0$
64. $-\frac{1}{2}\log|x^2 + xy + y^2| + \sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) = \log|x| + c$
65. $\log\left(\frac{y}{x}\right) = cy + 1$
66. $xy' - 2y = 0$

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