Differentiation

1/2 Marks

Differentiate each of the following w.r.t. x

1. $\sqrt{e^{\sqrt{x}}}$
2. $\cos x^3\left[\sin^2\left(x^5\right)\right]$
3. $\log\sqrt{\frac{1+x\cos x}{1-x\cos x}}$
4. $\log\left\{\sin\left(\log x\right)\right\}$
5. $\log\left\{\log\left(\log x^5\right)\right\}$
6. $\cos^{-1}\left[\frac{x-x^{-1}}{x+x^{-1}}\right]$
7. $\sin^{-1}\left(x\sqrt{x}\right)$
8. $\tan^{-1}\sqrt{\frac{1-x}{1+x}}$
9. $\cot^{-1}\left(\frac{1-x}{1+x}\right)$
10. $\tan^{-1}\left[\frac{\cos x-\sin x}{\cos x+\sin x}\right]$
11. $\sin^{-1}\sqrt{\frac{1+x}{2}}$
12. $\tan^{-1}\left(\frac{x^{\frac{1}{3}}+a^{\frac{1}{3}}}{1-x^{\frac{1}{3}}.a^{\frac{1}{3}}}\right)$
13. Find the set of all points where the function $f(x)=2x|x|$ is differentiable.
14. If $f(x)=\begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix}$ then find $f'(x)$
15. Find $\frac{dy}{dx}$ when $y=a^x.x^a$
16. Write for what value of x, $f(x)=|3x+1|$ is not derivable.
17. What is the derivative of $f(x)=|x-1|$ at $x=1$.
18. What is the derivative of x^6 w.r.t. x^2
19. Given that $g(0)=7$ and $f(x)=x.g(x)$. Also $f'(x)$ and $g'(x)$ exists, then write value of $f'(0)$
20. Write the derivatives of the following functions.
    (i) $\log_2(2x-1)$ (ii) $e^{3\log x}$ (iii) $2^{\sqrt{x}}$ (iv) $\tan^{-1}x^2+\cot^{-1}x^2$
    (v) $\log_x 5 \quad x>0$ (vi) $\tan^{-1}\left(\frac{3x-4}{1+12x}\right)$ (vii) $\sin^{-1}\left(x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right)$
    (viii) $\sec^{-1}\left(\frac{x+1}{x-1}\right)+\sin^{-1}\left(\frac{x-1}{x+1}\right)$
21. Differentiate $\cos^{-1}\theta$ w.r.t. $\log(1+\theta)$
22. If $f(1)=4$, $f'(1)=2$, find the value of the derivative of $\log\left\{f\left(e^x\right)\right\}$ w.r.t. x at the point $x=0$.
23. If the derivative of $\tan^{-1}(a+bx)$ w.r.t x takes the value 1 at $x=0$, write the relationship between a and b.

4/6Marks

1. Differentiate $x^{\sin x}$ w.r.t. $(\sin x)^x$

2. If $x=\sqrt{a^{\sin^{-1}t}}$ and $y=\sqrt{a^{\cos^{-1}t}}$, show that $\frac{dy}{dx}=-\frac{y}{x}$

3. If $x^y=y^x$, then find $\frac{dy}{dx}$

4. If $x=a\left(\cos\theta+\log\tan\frac{\theta}{2}\right)$ and $y=a\sin\theta$, then show that $\left.\frac{dy}{dx}\right]_{\theta=\frac{\pi}{4}}=1$

5. If $\frac{x}{a}=\sin 2t(1+\cos 2t)$ and $\frac{y}{b}=\cos 2t(1-\cos 2t)$, then show that $\left.\frac{dy}{dx}\right]_{t=\frac{\pi}{4}}=\frac{b}{a}$

6. Prove that: $\frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]=\sqrt{a^2-x^2}$

7. If $y=2x\tan^{-1}x-\log(1+x^2)$, prove $\frac{dy}{dx}=2\tan^{-1}x$

8. If $y=(\cos x)^{(\cos x)^{(\cos x)^{\cdots\infty}}}$, then show that $\frac{dy}{dx}=\frac{y^2\tan x}{y\log\cos x-1}$

8. If $y=\sqrt{\log x+\sqrt{\log x+\sqrt{\log x+\ldots\infty}}}$, then show that $(2y-1)\frac{dy}{dx}=\frac{1}{x}$

9. If $f(x)=x\sqrt{x^2+1}+\log\left(x+\sqrt{x^2+1}\right)$, then show that $f'(x)=2\sqrt{x^2+1}$

10. If $x^y=e^{x-y}$, then prove that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$

11. If $y=A\cos nx+B\sin nx$, then show that $\frac{d^2y}{dx^2}+n^2y=0$.

12. If $y=\tan x$, then show that $\frac{d^2y}{dx^2}-2y\frac{dy}{dx}=0$

13. If $y=3e^{2x}+2e^{3x}$, then show that $\frac{d^2y}{dx^2}-5y\frac{dy}{dx}+6y=0$

14. If $y=e^{\sin x}$, then show that $\frac{d^2y}{dx^2}-\cos x\frac{dy}{dx}+y\sin x=0$

15. If $y=\log\left(x+\sqrt{1+x^2}\right)$, then show that $(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$

16. If $x=\sin t$ and $y=\sin pt$, then show that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+p^2y=0$

17. If $y=Ae^{mx}+Be^{nx}$, show that $\frac{d^2y}{dx^2}-(m+n)\frac{dy}{dx}+mny=0$.

18. If $y=e^{\tan^{-1}x}$, then show that $(1+x^2)y_2+(2x-1)y_1=0$

19. If $y=\sin^{-1}x$, then show that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=0$

20. If $y=(\tan^{-1}x)^2$, then show that $(1+x^2)^2y_2+2x(1+x^2)y_1=2$.

21. If $y=3\cos(\log x)+4\sin(\log x)$, then show that $x^2y_2+xy_1+y=0$.

22. If $x=\log r$ and $y=\frac{1}{r}$, then show that $\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$.

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23. If $y=\sin(\sin x)$, then show that $\frac{d^2y}{dx^2}+\tan x\frac{dy}{dx}+y\cos^2 x=0$.

24. If $y=\frac{x\sin^{-1}x}{\sqrt{1-x^2}}+\log\sqrt{1-x^2}$, prove that $\frac{dy}{dx}=\frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}$

25. If $y=\cos^{-1}\left(\frac{3+5\cos x}{5+3\cos x}\right)$, then prove that $\cos x=\frac{4-5y_1}{3y_1}$

26. If $y=P\left(x+\sqrt{x^2-1}\right)^n+Q\left(x-\sqrt{x^2-1}\right)^n$, then show that $(x^2-1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-n^2y=0$.

27. If $e^{x+y}=xy$, then show that $\frac{dy}{dx}=\frac{y(1-x)}{x(y-1)}$.

28. If $(a+bx)e^{y/x}=x$, then prove that $x^3\frac{d^2y}{dx^2}=\left(x\frac{dy}{dx}-y\right)^2$.

29. Find the value of the derivative of $y(\log_{\cos x}\sin x)(\log_{\sin x}\cos x)^{-1}+\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ at $x=\frac{\pi}{4}$.

30. If $x\sqrt{1+y}+y\sqrt{1+x}=0$, then show that $\frac{dy}{dx}=\frac{-1}{(1+x)^2}$.

31. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, then show that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

32. If $e^{y-x}=xy$, then show that $\frac{dy}{dx}=\frac{2-\log x}{(1-\log x)^2}$.

33. If $\sin^{-1}y=2\log(x+1)$, then show that $(x+1)^2y_2+(x+1)y_1+ay=0$.

34. If $y=\log\left\{\left(\frac{x}{a+bx}\right)^2\right\}$, then show that $x^3y_2=(xy_1-y)^2$

35. If $x=\sec\theta-\cos\theta$ and $y=\sec^n\theta-\cos^n\theta$, then prove that $(x^2+4)\left(\frac{dy}{dx}\right)^2=n^2(y^2+4)$.

36. If $x=\sin\left(\frac{1}{a}\log y\right)$, then show that $(1-x^2)y_2-xy_1-a^2y=0$.

37. If $x=\sqrt{a^{\sin^{-1}t}}$ and $y=\sqrt{a^{\cos^{-1}t}}$, show that $x\frac{d^2y}{dx^2}+2\frac{dy}{dx}=0$

38. If $y=\sqrt{9+\sqrt{9+\sqrt{9+x^2}}}$, then show that $\frac{dy}{dx}=\frac{x}{4\sqrt{9+\sqrt{9+\sqrt{9+x^2}}}.\sqrt{9+\sqrt{9+x^2}}.\sqrt{9+x^2}}$.

39. Given that $\cos\frac{x}{2}.\cos\frac{x}{4}.\cos\frac{x}{8}.....=\frac{\sin x}{x}$, then prove that $\frac{1}{2^2}\sec^2\frac{x}{2}+\frac{1}{2^4}\sec^2\frac{x}{4}+....=\operatorname{cosec}^2x-\frac{1}{x^2}$.

40. If $y=\sec x-\tan x$, show that $\cos x\frac{d^2y}{dx^2}=y^2$.

41. If $y=\tan x+\sec x$, then prove that $\frac{d^2y}{dx^2}=\frac{\cos x}{(1-\sin x)^2}$

42. If $x^p.y^q=(x+y)^{p+q}$, then prove that $\frac{dy}{dx}=\frac{y}{x}$.

43. If $y=\log\left\{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right\}$, show that $\frac{dy}{dx}-\sec x=0$.

44. If $y=\left\{x+\sqrt{x^2+a^2}\right\}^n$, then prove that $\frac{dy}{dx}=\frac{ny}{\sqrt{x^2+a^2}}$

45. If $y=\sqrt{x}+\frac{1}{\sqrt{x}}$, then show that $2x\frac{dy}{dx}+y=2\sqrt{x}$.

46. If $y=\sin\left(\log x\right)$, prove that $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$.

47. If $(x-a)^2+(y-b)^2=c^2$ for some $c>0$, prove that $\frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$, is a constant independent of a and b.

48. If $y=e^{a\cos^{-1}x}$, $-1\le x\le 1$. Show that $\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-a^2y=0$.

49. If $y=\cfrac{\sin x}{1+\cfrac{\cos x}{1+\cfrac{\sin x}{1+\cfrac{\cos x}{1+\cfrac{\sin x}{1+....\infty}}}}}$, prove that $\frac{dy}{dx}=\frac{(1+y)\cos x+y\sin x}{(1+2y)+\cos x-\sin x}$

50. If $y=x\log\left[(ax)^{-1}+a^{-1}\right]$, prove that $x(x+1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=y-1$

51. If $y=\sin^{-1}\left[x^2\sqrt{1-x^2}+x\sqrt{1-x^4}\right]$, the prove that $\frac{dy}{dx}=\frac{2x}{\sqrt{1-x^4}}+\frac{1}{\sqrt{1-x^2}}$.

52. If $y=\frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\left[\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right]$, prove that $\frac{dy}{dx}=\frac{1}{a+b\cos x}$, $a>b>0$.

53. If $y=\tan^{-1}\left[\frac{4x}{1+5x^2}\right]+\tan^{-1}\left[\frac{2+3x}{3-2x}\right]$, show that $\frac{dy}{dx}=\frac{5}{1+25x^2}$.

54. Prove that $\frac{d}{dx}\left[\frac{1}{4\sqrt{2}}\log\left|\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}\right|+\frac{1}{2\sqrt{2}}\tan^{-1}\frac{\sqrt{2}x}{1-x^2}\right]=\frac{1}{1+x^4}$

55. If $\frac{1}{\sqrt{b^2-a^2}}\log\left[\frac{\sqrt{b+a}+\sqrt{b-a}\tan\frac{x}{2}}{\sqrt{b+a}-\sqrt{b-a}\tan\frac{x}{2}}\right]$, prove that $\frac{dy}{dx}=\frac{\sec^2\frac{x}{2}}{(b+a)-(b-a)\tan^2\frac{x}{2}}$

56. If $3^x+3^y=3^{x+y}$, then prove that $\frac{dy}{dx}=-3^{y-x}$.

57. If $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then show that $\frac{d^2y}{dx^2}=\frac{-b^4}{a^2y^3}$.

58. If $y^3=3ax^2-x^3$, then prove that $\frac{d^2y}{dx^2}=\frac{-2a^2x^2}{y^5}$.

Differentiate each of the following w.r.t x.(59 to 67)

59. x^{x^x} 60. $x^{\sin x}(\sin x)^x$ 61. $\frac{8^x}{x^8}$

62. $x = a\left(\theta + \frac{1}{\theta}\right),\ y = a\left(\theta - \frac{1}{\theta}\right)$

63. $\frac{(x+1)^{3/4}\, e^{mx} \sin^{-1}(ax)}{(x-1)^{7/3}(x+2)^{5/7}}$

64. $(\log x)^x + x^{\log x}$

65. $\tan^{-1}\left(\frac{x}{1+6x^2}\right)$

66. $\cos^{-1}\left(\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right)$

67. $\sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{\sqrt{13}}\right)$

68. If $\sqrt{1-x^8} + \sqrt{1-y^8} = a\left(x^4 - y^4\right)$, show that $\frac{dy}{dx} = \frac{x^3}{y^3}\sqrt{\frac{1-y^8}{1-x^8}}$.

69. If $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$, prove that $\frac{dy}{dx} = -\frac{y \log x}{x \log y}$.

70. If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$, prove that $\frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right)$.

71. If $y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{x-c} + 1$, prove that $\frac{dy}{dx} = \frac{y}{x}\left\{\frac{a}{(a-x)} + \frac{b}{(b-x)} + \frac{c}{c-x}\right\}$

72. Find A and B so that $y = A\sin 3x + B\cos 3x$ satisfies the equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 10\cos 3x$.

73. If $y = Ae^{-kt}\cos(pt + c)$, prove that $\frac{d^2y}{dt^2} + 2k\frac{dy}{dt} + n^2 y = 0$, where $n^2 = p^2 + k^2$.

74. Differentiate $x^{\sin x}$ w.r.t. $(\sin x)^x$

75. Differentiate $\cos^{-1}\left[\frac{3\cos x - 2\sin x}{\sqrt{13}}\right]$ w.r.t. $\sin^{-1}\left[\frac{5\sin x + 4\cos x}{\sqrt{41}}\right]$.

Verify Rolle's Theorem for each of the following. (76 to 78)

76. $f(x) = x^3 - 7x^2 + 16x - 12$ in $[2,3]$

77. $f(x) = \sin x - \sin 2x, [0,\pi]$

78. $f(x) = \sin^4 x + \cos^4 x, \left[0, \frac{\pi}{2}\right]$

79. Using Rolle's theorem, find points on the curve $y = 16 - x^2$, $x \in [-1,1]$, where tangent is parallel to x-axis.

80. It is given that for the function $f(x) = x^3 - 6x^2 + ax + b$ on $[1,3]$, Rolle's theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$. Find the values of a and b if $f(1) = f(3) = 0$.

Verify MVT for the following function

81. $f(x) = (x-1)(x-2)(x-3)$ in $[0,4]$

82. $f(x) = 2\sin x + \sin 2x$ on $[0,\pi]$

83. Using MVT , find a point on the curve $y = \sqrt{x-2}$ defined on the interval $[2,3]$, where the tangent is parallel to the chord joining the end points of the curve.

ANSWERS

1 Mark

1. $\frac{1}{4\sqrt{x}}e^{\frac{1}{2}\sqrt{x}}$
2. $10x^4\sin x^5.\cos x^5.\cos x^3-3x^2.\sin x^3.\sin^2 x^5$
3. $\frac{\cos x-x\sin x}{1-x^2\cos^2 x}$
4. $\frac{\cot(\log x)}{x}$
5. $\frac{5}{x\log\{\log x^5\}\log x^5}$
6. $\frac{-2}{1+x^2}$
7. $\frac{3}{2}\sqrt{\frac{x}{1-x^3}}$
8. $\frac{-1}{2\sqrt{1-x^2}}$
9. $\frac{1}{1+x^2}$
10. -1
11. $\frac{1}{2\sqrt{1-x^2}}$
12. $\frac{1}{3x^{2/3}\left(1+x^{2/3}\right)}$
13. $(-\infty,\infty)$
14. $6x^2$
15. $x^a a^x\log a+a^{x+1}x^{a-1}$
16. $x=-\frac{1}{3}$
17. Derivative is not exists
18. $3x^4$
19. 7
20. (i) $\frac{2}{(2x-1)\log 2}$ (ii) $3x^2$ (iii) $\frac{2^{\sqrt{x}}\log 2}{2\sqrt{x}}$ (iv) 0 (v) $\frac{-1}{(\log_5 x)^2.x\log 5}$ (vi) $\frac{3}{1+9x^2}$ (vii) $\frac{1}{\sqrt{1-x^2}}-\frac{1}{2\sqrt{x-x^2}}$ (viii) 0
21. $-\sqrt{\frac{1+\theta}{1-\theta}}$
22. $\frac{1}{2}$
23. $1+a^2=b$

4/6Marks

1. $\frac{x^{\sin x}\left(\frac{\sin x}{x}+\log\cos x\right)}{(\sin x)^x\left(x\cot x+\log\sin x\right)}$
3. $\frac{y}{x}\left(\frac{y-x\log y}{x-y\log x}\right)$
29. $\frac{-8}{\log 2}+\frac{32}{16+\pi^2}$
59. $x^{x^x}.x^x\left[\frac{1}{x}+\log x(1+\log x)\right]$
60. $x^{\sin x}(\sin x)^x\left[\cos x\log x+\frac{\sin x}{x}+\log\sin x+\cot x\right]$
61. $\frac{8^x(x\log x-8)}{x^9}$
62. $\frac{\theta^2+1}{\theta^2-1}$
63. $\frac{(x+1)^{3/4}e^{mx}\sin^{-1}(ax)}{(x-1)^{7/3}(x+2)^{5/7}}\left[\frac{3}{4(x+1)}+m+\frac{a}{\sqrt{1-a^2x^2}\sin^{-1}(ax)}-\frac{7}{3(x-1)}-\frac{5}{7(x+2)}\right]$
64. $(\log x)^{x-1}\left[1+\log x.\log(\log x)\right]+2x^{\log x-1}.\log x$
65. $\frac{3}{1+9x^2}-\frac{2}{1+4x^2}$
66. $-\frac{1}{\sqrt{1-x^2}}$
67. $\frac{1}{\sqrt{1-x^2}}$
72. $A=\frac{2}{3},B=-\frac{1}{3}$
75. 1
79. $(0,16)$

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80. $a = 11, b = -6$ 83. $\left(9/4, 1/2\right)$

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