

# Indefinite Integration

## Introduction

We have discussed methods for finding derivatives of functions in previous chapters. We will now turn our attention to reversing the operation of differentiation. Given the derivative of a function, we will attempt to find the function. This process is called anti-derivative of a function. For example, if the derivative of a function is  $2x$ , we know the function could be  $f(x) = x^2$  because  $\frac{d}{dx}(x^2) = 2x$ . But the function could also be

$f(x) = x^2 + 4$  because  $\frac{d}{dx}(x^2 + 4) = 2x$ . It is clear that any function of the form

$f(x) = x^2 + c$ , Where  $c$  is a constant, will have  $f'(x) = 2x$  as its derivative. Thus, we say the anti-derivative of  $f'(x) = 2x$  is  $f(x) = x^2 + c$ , where  $c$  is any arbitrary constant. This process of finding an anti-derivative is called *integration*. In this chapter, we study some methods of integration.

## Mathematical Notation

If the differential coefficient of  $F(x)$  is  $f(x)$

i.e. 
$$\frac{d}{dx}[F(x)] = f(x)$$

then we say that the anti-derivative or integral of  $f(x)$ , written as  $\int f(x)dx = F(x)$

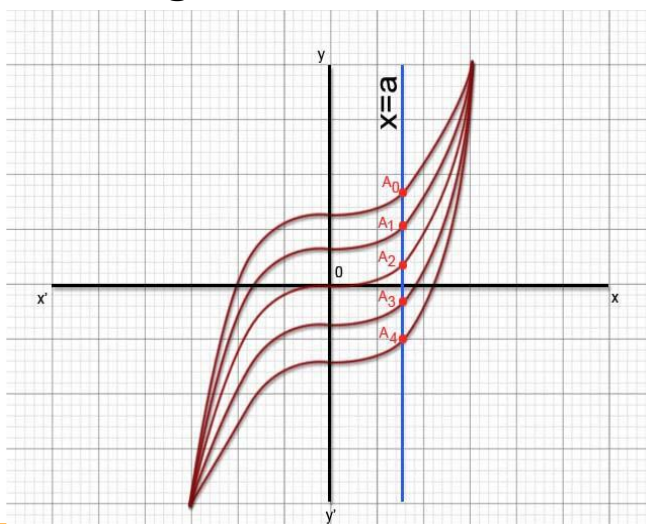
Also for any constant  $c$ , 
$$\frac{d}{dx}[F(x) + c] = f(x)$$

$\therefore \int f(x)dx = F(x) + c$

Where  $c$  is an arbitrary constant.

This shows that  $F(x)$  and  $F(x) + c$  are both integrals of the same function  $f(x)$ . Thus for different values of  $c$ , we obtain different integrals of  $f(x)$  i.e. the integral of  $f(x)$  is not definite. By virtue of this property,  $F(x)$  is called the indefinite integral of  $f(x)$ .

## Graphical Meaning



### List of Formulae

- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$
- $\int e^x dx = e^x + c$
- $\int k dx = kx + c$
- $\int \frac{1}{x} dx = \log_e |x| + c$
- $\int \sin x dx = -\cos x + c$
- $\int \cos x dx = \sin x + c$
- $\int \sec^2 x dx = \tan x + c$
- $\int \operatorname{cosec}^2 x dx = -\cot x + c$
- $\int \sec x \cdot \tan x dx = \sec x + c$
- $\int \operatorname{cosec} x \cdot \cot x dx = -\operatorname{cosec} x + c$
- $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$
- $\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + c$
- $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$
- $\int \frac{-1}{1+x^2} dx = \cot^{-1} x + c$

- $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$
- $\int \frac{-1}{x\sqrt{x^2-1}} dx = \operatorname{cosec}^{-1} x + c$
- $\int \tan x dx = -\log_e |\cos x| + c$   
Or  
 $\int \tan x dx = \log_e |\sec x| + c$
- $\int \cot x dx = \log_e |\sin x| + c$   
Or  
 $\int \cot x dx = -\log_e |\operatorname{cosec} x| + c$
- $\int \sec x dx = \log_e |\sec x + \tan x| + c$
- $\int \operatorname{cosec} x dx = \log_e |\operatorname{cosec} x - \cot x| + c$
- $\int a^x dx = \frac{a^x}{\log_e a} + c$

### Some standard results on integration:

- $\frac{d}{dx} \left( \int f(x) dx \right) = f(x)$
- $\int k f(x) dx = k \int f(x) dx$ , where k is a constant
- $\int \{f(x) \pm g(x)\} dx = \int f(x) dx \pm \int g(x) dx$
- $\int \{k_1 f_1(x) \pm k_2 f_2(x) \pm \dots \pm k_n f_n(x)\} dx = k_1 \int f_1(x) dx \pm k_2 \int f_2(x) dx \pm \dots \pm k_n \int f_n(x) dx$

### Example Exercise:

- $\int \{ e^{x \log a} + e^{\log x} + e^{\frac{1}{\log a}} \} dx$
- $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$
- $\int \frac{1}{1 + \sin x} dx$
- $\int \frac{2^x + 3^x}{5^x} dx$
- $\int \frac{x^4 + x^2 + 1}{x^2 - x + 1} dx$

### Answers

- $\frac{a^x}{\log a} + \frac{x^{a+1}}{a+1} + a^a \cdot x + C$
- $2 \sin x + 2x \cos \alpha + C$
- $\tan x - \sec x + C$
- $\frac{\left(\frac{2}{5}\right)^x}{\log_e \frac{2}{5}} + \frac{\left(\frac{3}{5}\right)^x}{\log_e \frac{3}{5}} + C$
- $\frac{x^3}{3} + \frac{x^2}{2} + x + C$

### Practice Exercise: 1

- $\int \frac{x^3 - 1}{x^2} dx$
- $\int (x^{\frac{2}{3}} + 1) dx$
- $\int (x^{\frac{3}{2}} + 2e^x - \frac{1}{x}) dx$
- $\int (\sin x + \cos x) dx$
- $\int \operatorname{cosec} x (\operatorname{cosec} x + \cot x) dx$
- $\int \frac{1 - \sin x}{\cos^2 x} dx$
- $\int x^2 \left(1 - \frac{1}{x^2}\right) dx$
- $\int (ax^2 + bx + c) dx$
- $\int (2x^2 + e^x) dx$
- $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$
- $\int \frac{x^3 + 5x^2 - 4}{x^2} dx$
- $\int \frac{x^3 + 3x + 4}{\sqrt{x}} dx$

13.  $\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$
14.  $\int (1 - x)\sqrt{x} dx$
15.  $\int (2x - 3 \cos x + e^x) dx$
16.  $\int (2x^2 - 3 \sin x + 5\sqrt{x}) dx$
17.  $\int \sec x (\sec x + \tan x) dx$
18.  $\int \frac{\sec^2 x}{\cos e^{2x}} dx$
19.  $\int \frac{2 - 3 \sin x}{\cos^2 x} dx$
20.  $\int \frac{1}{\sin^2 x \cos^2 x} dx$
21.  $\int \sqrt{1 - \cos 2x} dx$
22.  $\int \sqrt{1 + \sin 2x} dx$
23.  $\int \frac{\sec x}{\sec x + \tan x} dx$
24.  $\int \sin^{-1}(\cos x) dx$

25.  $\int \tan^{-1} \left\{ \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}} \right\} dx$
26.  $\int \frac{x^4}{x^2 + 1} dx$
27.  $\int \frac{(a^x + b^x)^2}{a^x b^x} dx$
28.  $\int \frac{e^{5 \log_e x} - e^{4 \log_e x}}{e^{3 \log_e x} - e^{2 \log_e x}} dx$
29.  $\int \sin(e^x) d(e^x)$
30.  $\int a^x e^x dx$
31. If  $f'(x) = e^x + \frac{1}{1+x^2}$  and  $f(1) = 0$ .  
Then find  $f(x)$ .
32. If  $f'(x) = a \sin x + b \cos x$  and  
 $f'(0) = 4$ ,  $f(0) = 3$ ,  $f\left(\frac{\pi}{2}\right) = 5$ ,  
find  $f(x)$ .

**Integral of the form :**  $\int f(ax + b) dx$

If  $\int f(x) dx = g(x) + C$ , then  $\int f(ax + b) dx = \frac{1}{a} g(ax + b) + C$

**Example Exercise: 2**

1.  $\int \frac{1}{\sqrt{x+3} - \sqrt{x+2}} dx$
2.  $\int a^{3x+2} dx$
3.  $\int \sin^3(2x+1) dx$

**Answers**

1.  $\frac{2}{3} \left[ (x+3)^{3/2} + (x+2)^{3/2} \right] + C$
2.  $\frac{a^{3x+2}}{3 \log a} + C$
3.  $-\frac{3}{8} \cos(2x+1) + \frac{1}{24} \cos(6x+3) + C$

**Practice Exercise: 2**

**Integral of the form:**

$\int \sin^m x dx$ ,  $\int \cos^m x dx$  where  $m \leq 4$ , then use trigonometric identities

- (i)  $\sin^2 x = \frac{1 - \cos 2x}{2}$       (ii)  $\cos^2 x = \frac{1 + \cos 2x}{2}$       (iii)  $\sin 3x = 3 \sin x - 4 \sin^3 x$       (iv)  $\cos 3x = 4 \cos^3 x - 3 \cos x$

### Example Exercise: 3

- $\int \sin^4 x dx$
- $\int \sin^3 x \cos^3 x dx$
- $\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx$

#### Answers

- $\frac{1}{8} \left[ 3x - 2\sin 2x + \frac{\sin 4x}{4} \right] + C$
- $\frac{1}{32} \left[ -\frac{3}{2} \cos 2x + \frac{1}{6} \cos 6x \right] + C$
- $-\frac{1}{2} \sin 2x + C$

### Practice Exercise: 3

#### Find the following integrals:

- $\int \sin^2 x dx$
- $\int \cos^2 x dx$
- $\int \cos^3 x dx$
- $\int \cos^4 x dx$
- $\int \sin^4 x \cos^4 x dx$
- $\int \cos^4 2x dx$
- $\int \sin^2 bx dx$
- $\int \sin^2 \frac{x}{2} dx$
- $\int \frac{1 + \cos x}{1 - \cos x} dx$

#### Integral of the form:

$\int \sin mx \cos nx dx$ ,  $\int \sin mx \sin nx dx$ ,  $\int \cos mx \cos nx dx$  and  $\int \cos mx \sin nx dx$  then use trigonometric identities :

- (i)  $2\sin A \cos B = \sin(A+B) + \sin(A-B)$  (ii)  $2\cos A \sin B = \sin(A+B) - \sin(A-B)$   
 (iii)  $2\cos A \cos B = \cos(A+B) + \cos(A-B)$  (iv)  $2\sin A \sin B = \cos(A-B) - \cos(A+B)$

### Example Exercise: 4

- $\int \cos 2x \cos 4x dx$
- $\int \sin x \sin 2x \sin 3x dx$
- $\int \frac{1 + \cos 4x}{\cot x - \tan x} dx$

#### Answers

- $\frac{1}{2} \left\{ \frac{\sin 6x}{6} + \frac{\sin 2x}{2} \right\} + C$
- $\frac{1}{4} \left\{ -\frac{\cos 4x}{4} - \frac{\cos 2x}{2} + \frac{\cos 6x}{6} \right\} + C$
- $-\frac{1}{8} \cos 4x + C$

**Integral of the form:**  $\int \frac{f'(x)}{f(x)} dx = \log_e |f(x)| + C$

Proof: Let  $I = \int \frac{f'(x)}{f(x)} dx$

putting  $f(x) = t$  and  $f'(x) dx = dt$  then we have

$$I = \int \frac{1}{t} dt = \log_e |t| + C = \log_e |f(x)|$$

### Practice Exercise: 4

#### Find the following integrals:

- $\int \sin 3x \cos 4x dx$
- $\int \cos 2x \cos 4x \cos 6x dx$
- $\int \frac{\sin 4x}{\cos 2x} dx$
- $\int \frac{\sin 4x}{\sin x} dx$
- $\int \sqrt{17} \sin x dx$
- $\int \sin mx \cos nx dx$

### Example Exercise: 5

- $\int \frac{1}{x \log x \log(\log x)} dx$
- $\int \frac{1}{\sin x \cos^3 x} dx$

3.  $\int x^5 \sqrt{a^3 + x^3} dx$

4.  $\int \frac{\operatorname{cosec} x}{\log \tan \frac{x}{2}} dx$

**Answers**

1.  $\log \{ \log(\log x) \} + C$

2.  $\frac{1}{2} \tan^2 x + \log |\tan x| + C$

3.  $\frac{2}{15} (a^3 + x^3)^{5/2} - \frac{2}{9} a^3 (x^3 + a^3)^{3/2} + C$

4.  $\log \left| \log \tan \frac{x}{2} \right| + C$

**Practice Exercise: 5**

Find the following integrals:

1.  $\int 2x \sin(x^2 + 1) dx$

2.  $\int \frac{\tan^4 \sqrt{x} \sec^2 \sqrt{x}}{\sqrt{x}} dx$

3.  $\int \frac{\sin(\tan^{-1} x)}{1+x^2} dx$

4.  $\int \frac{\sin x}{\sin(x+a)} dx$

5.  $\int \frac{1}{1+\tan x} dx$

6.  $\int \frac{2x}{1+x^2} dx$

7.  $\int \frac{1}{x+x \log x} dx$

8.  $\int \sin x \sin(\cos x) dx$

9.  $\int x \sqrt{1+2x^2} dx$

10.  $\int (4x+2) \sqrt{x^2+x+1} dx$

11.  $\int \frac{1}{x-\sqrt{x}} dx$

12.  $\int \frac{x}{9-4x^2} dx$

13.  $\int \frac{x}{e^{x^2}} dx$

14.  $\int \frac{e^{\tan^{-1} x}}{1+x^2} dx$

15.  $\int \frac{e^{2x}-1}{e^{2x}+1} dx$

16.  $\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}} dx$

17.  $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$

18.  $\int \frac{2 \cos x - 3 \sin x}{6 \cos x + 4 \sin x} dx$

19.  $\int \frac{1}{\cos^2 x (1 - \tan x)^2} dx$

20.  $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$

21.  $\int \frac{\cos x}{\sqrt{1+\sin x}} dx$

22.  $\int \cot x \log \sin x dx$

23.  $\int \frac{\sin x}{1+\cos x} dx$

24.  $\int \frac{1}{1+\cot x} dx$

25.  $\int \frac{1}{1-\tan x} dx$

26.  $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$

27.  $\int \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8} dx$

28.  $\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx$

29.  $\int \frac{1}{1-e^{-x}} dx$

30.  $\int \frac{\sin 2x}{a^2 \sin^2 x + b^2 \cos^2 x} dx$

31.  $\int \frac{\sec x \cos ecx}{\log(\tan x)} dx$

32.  $\int \frac{a}{b+ce^x} dx$

33.  $\int \frac{1}{1+e^x} dx$

34.  $\int \frac{\cos ecx}{\log \tan \frac{x}{2}} dx$

35.  $\int \frac{\sec x}{\log(\sec x + \tan x)} dx$

36.  $\int \frac{1}{x \log x \log(\log x)} dx$

37.  $\int \frac{e^{x-1} + x^{e-1}}{e^x + x^e} dx$

38.  $\int \frac{x+1}{x(x+\log x)} dx$



**Integral of the form:**  $\int \{f(x)\}^n f'(x) dx$

$$\int \{f(x)\}^n f'(x) dx = \frac{\{f(x)\}^{n+1}}{n+1} + C ;$$

Proof: Let  $I = \int \{f(x)\}^n f'(x) dx$

putting  $f(x) = t$  and  $f'(x) dx = dt$ , we get

$$I = \int t^n dt = \frac{t^{n+1}}{n+1} + C = \frac{\{f(x)\}^{n+1}}{n+1} + C$$

### Example Exercise: 6

1.  $\int \frac{\{e^{\sin^{-1}x}\}^2}{\sqrt{1-x^2}} dx$
2.  $\int \frac{\sec^2(2\tan^{-1}x)}{1+x^2} dx$
3.  $\int \sec^3 x \tan x dx$
4.  $\int \frac{(x^4 - x)^{1/4}}{x^5} dx$
5.  $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$

### Answers

1.  $\frac{\{e^{\sin^{-1}x}\}^2}{2} + C$
2.  $\frac{1}{2} \tan(2\tan^{-1}x) + C$
3.  $\frac{\sec^3 x}{3} + C$
4.  $\frac{4}{15} \left(1 - \frac{1}{x^3}\right)^{5/4} + C$

### Practice Exercise: 6

Find the following integrals:

1.  $\int \frac{(\log x)^2}{x} dx$
2.  $\int (x^3 - 1)^{1/3} x^5 dx$
3.  $\int \frac{x^2}{(2+3x^3)^3} dx$
4.  $\int \frac{1}{x(\log x)^m} dx, x > 0$
5.  $\int \sqrt{\sin 2x} \cos 2x dx$
6.  $\int \frac{\sin x}{(1+\cos x)^2} dx$
7.  $\int \frac{(1+\log x)^2}{x} dx$
8.  $\int \frac{(x+1)(x+\log x)^2}{x} dx$
9.  $\int \frac{4(\sin^{-1} x)^3}{\sqrt{1-x^2}} dx$
10.  $\int \frac{e^x}{(1+e^x)^2} dx$
11.  $\int \frac{\{e^{\sin^{-1}x}\}^2}{\sqrt{1-x^2}} dx$
12.  $\int \tan^{3/2} x \sec^2 x dx$

**Integral of the form:**  $\int \sin^m x \cos^n x dx$ , to evaluate use the following steps:

- (i) If m is odd positive integer and n is even positive integer then put  $\cos x = t$
- (ii) If m is even positive integer and n is odd positive integer then put  $\sin x = t$ .
- (iii) If m and n both are odd positive integer then put  $\sin x$  or  $\cos x$  which has higher odd positive power.

### Example Exercise: 7

1.  $\int \sin^3 x \cos^6 x \, dx$
2.  $\int x \cos^3 x^2 \sin x^2 \, dx$
3.  $\int \cos^7 x \, dx$

#### Answers

1.  $-\left\{ \frac{\cos^7 x}{7} - \frac{\cos^9 x}{9} \right\} + C$
2.  $-\frac{1}{8} \cos^4 x^2 + C$

**Integral of the form:**  $\int \frac{1}{ax^2 + bx + c} \, dx$ , to evaluate use the following steps:

- (i) Make coefficient of  $x^2$  unity, if it is not, by multiplying and dividing by it.
- (ii) Add and subtract the square of half of the coefficient of  $x$  to the express

$$ax^2 + bx + c \text{ in the form of } \left[ \left( x + \frac{b}{2a} \right)^2 + \frac{4ac - b^2}{4a^2} \right]$$

- (iii) Use the suitable formula from the following formulas:

- (a)  $\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + C$
- (b)  $\int \frac{1}{a^2 - x^2} \, dx = \frac{1}{2a} \log_e \left| \frac{x+a}{x-a} \right| + C$
- (c)  $\int \frac{1}{x^2 - a^2} \, dx = \frac{1}{2a} \log_e \left| \frac{x-a}{x+a} \right|$

### Example Exercise: 8

1.  $\int \frac{1}{9x^2 + 6x + 10} \, dx$
2.  $\int \frac{1}{1 + x - x^2} \, dx$
3.  $\int \frac{1}{x(x^n - 1)} \, dx$

#### Answers

1.  $\frac{1}{9} \tan^{-1} \left( \frac{3x+1}{3} \right) + C$
2.  $\frac{1}{\sqrt{5}} \log \left| \frac{\sqrt{5}-1+2x}{\sqrt{5}+1-2x} \right| + C$
3.  $\frac{1}{n} \log \left| \frac{x^n}{x^n + 1} \right| + C$

$$3. \sin x - \sin^3 x + \frac{3}{5} \sin^5 x - \frac{1}{7} \sin^7 x + C$$

### Practice Exercise: 7

**Find the following integrals:**

1.  $\int \sin^5 x \, dx$
2.  $\int \sin^4 x \cos^3 x \, dx$
3.  $\int \sin^7 x \, dx$
4.  $\int \sin^3 x \cos^5 x \, dx$

### Practice Exercise: 8

**Find the following integrals:**

1.  $\int \frac{dx}{x^2 - 16}$
2.  $\int \frac{dx}{x^2 - 6x + 13}$
3.  $\int \frac{dx}{3x^2 + 13x - 10}$
4.  $\int \frac{3x^2}{x^6 + 1} \, dx$
5.  $\int \frac{3x}{1 + 2x^4} \, dx$
6.  $\int \frac{x^2}{1 - x^6} \, dx$
7.  $\int \frac{1}{9x^2 + 6x + 5} \, dx$

8.  $\int \frac{1}{x^2 + 2x + 2} dx$

**Integral of the form:**  $\int \frac{1}{\sqrt{ax^2 + bx + c}} dx$ , to evaluate use the following steps:

- (i) Make coefficient of  $x^2$  unity, if it is not, by multiplying and dividing by it.
- (ii) Add and subtract the square of half of the coefficient of  $x$  to the express

$ax^2 + bx + c$  in the form of  $\left[ \left( x + \frac{b}{2a} \right)^2 + \frac{4ac - b^2}{4a^2} \right]$  or  $\left[ \frac{4ac - b^2}{4a^2} - \left( x + \frac{b}{2a} \right)^2 \right]$

- (iii) Use the suitable formula from the following formulas:

(a)  $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left( \frac{x}{a} \right) + C$

(b)  $\int \frac{1}{\sqrt{a^2 + x^2}} dx = \log_e \left| x + \sqrt{a^2 + x^2} \right| + C$

(c)  $\int \frac{1}{\sqrt{x^2 - a^2}} dx = \log_e \left| x + \sqrt{x^2 - a^2} \right| + C$

**Example Exercise: 9**

1.  $\int \frac{1}{\sqrt{x(1-2x)}} dx$

2.  $\int \frac{1}{\sqrt{(x-\alpha)(\beta-x)}} dx$

3.  $\int \sqrt{\frac{x}{a^3 - x^3}} dx$

4.  $\int \sqrt{\sec x - 1} dx$

5.  $\int \frac{\sin(x-\alpha)}{\sqrt{\sin(x+\alpha)}} dx$

6.  $\int \frac{1}{x^{2/3} \sqrt{x^{2/3} - 4}} dx$

**Answers**

1.  $\frac{1}{\sqrt{2}} \sin^{-1}(4x-1) + C$

2.  $2 \sin^{-1} \left( \sqrt{\frac{x-\alpha}{\beta-\alpha}} \right) + C$

3.  $\frac{2}{3} \sin^{-1} \left( \frac{x^{3/2}}{a^{3/2}} \right) + C$

4.  $-\log \left| \left( \cos x + \frac{1}{2} \right) + \sqrt{\cos^2 x + \cos x} \right| + C$

5.  $-\cos \alpha \sin^{-1} \left( \frac{\cos x}{\cos \alpha} \right) - \sin \alpha \cdot \log \left| \sin x + \sqrt{\sin^2 x - \sin^2 \alpha} \right| + C$

6.  $3 \log \left| x^{1/3} + \sqrt{x^{2/3} - 4} \right| + C$

**Practice Exercise: 9**

Find the following integrals:

1.  $\int \frac{dx}{\sqrt{2x-x^2}}$

2.  $\int \frac{dx}{\sqrt{5x^2-2x}}$

3.  $\int \frac{dx}{\sqrt{1+4x^2}}$

4.  $\int \frac{dx}{\sqrt{(2-x)^2+1}}$

5.  $\int \frac{x-1}{\sqrt{x^2-1}} dx$

6.  $\int \frac{x^2}{\sqrt{x^6+a^6}} dx$

7.  $\int \frac{\sec^2 x}{\sqrt{\tan^2 x+4}} dx$

8.  $\int \frac{1}{\sqrt{x^2+2x+2}} dx$

9.  $\int \frac{1}{\sqrt{(x-1)(x-2)}} dx$

10.  $\int \frac{1}{\sqrt{(x-a)(x-b)}} dx$

**Integral of the form:**  $\int \frac{px+q}{ax^2+bx+c} dx$ , To evaluate this type of integral use the following steps:

- (i) Let  $px + q = A \frac{d}{dx}(ax^2 + bx + c) + B$



$$\Rightarrow px + q = A(2ax + b) + B \dots \dots \dots (1)$$

- (ii) Obtain the values of A and B by equating the coefficient of like powers of x on both sides
- (iii) Put the values of A and B in equation (1) dividing by  $ax^2 + bx + c$  on both side of equation (1) and taking integration on both the side. then we get

$$\int \frac{px + q}{ax^2 + bx + c} dx = A \int \frac{2ax + b}{ax^2 + bx + c} dx + B \int \frac{1}{ax^2 + bx + c} dx$$

- (iv) Integrate R.H.S. of step (iii)

### Example Exercise: 10

1.  $\int \frac{1-3x}{3x^2+4x+2} dx$
2.  $\int \frac{2\sin 2x - \cos x}{6 - \cos^2 x - 4\sin x} dx$

### Answers

1.  $-\frac{1}{2} \log|3x^2 + 4x + 2| + \frac{3}{\sqrt{2}} \tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right) + C$
2.  $2 \log|\sin^2 x - 4\sin x + 5| + 7 \tan^{-1}(\sin x - 2) + C$



### Practice Exercise: 10

**Find the following integrals:**

1.  $\int \frac{x+2}{2x^2+6x+5} dx$
2.  $\int \frac{x+3}{x^2-2x-5} dx$
3.  $\int \frac{5x-2}{1+2x+3x^2} dx$
4.  $\int \frac{x^3+x}{x^4-9} dx$
5.  $\int \frac{1}{2e^{2x}+3e^x+1} dx$
6.  $\int \frac{(3\sin x - 2)\cos x}{5 - \cos^2 x - 4\sin x} dx$

**Integral of the form:**  $\int \frac{px + q}{\sqrt{ax^2 + bx + c}} dx$ , To evaluate this type of integral use the

following steps:

- (i) Let  $px + q = A \frac{d}{dx}(ax^2 + bx + c) + B$   
 $\Rightarrow px + q = A(2ax + b) + B \dots \dots \dots (1)$
- (ii) Obtain the values of A and B by equating the coefficient of like powers of x on both sides
- (iii) Put the values of A and B in equation (1) dividing by  $\sqrt{ax^2 + bx + c}$  on both sides of equation (1) and taking integration on both the sides. then we get :

$$\int \frac{px + q}{\sqrt{ax^2 + bx + c}} dx = A \int \frac{2ax + b}{\sqrt{ax^2 + bx + c}} dx + B \int \frac{1}{\sqrt{ax^2 + bx + c}} dx$$

- (iv) Integrate R.H.S. of step (iii)

### Example Exercise: 11

1.  $\int \frac{2x+3}{\sqrt{x^2+4x+5}} dx$
2.  $\int \sqrt{\frac{1+x}{x}} dx$

### Answers

1.  $2\sqrt{x^2+4x+5} - \log|x+2+\sqrt{x^2+4x+5}| + C$
2.  $\sqrt{x^2+x} + \frac{1}{2} \log\left|x + \frac{1}{2} + \sqrt{x^2+x}\right| + C$

### Practice Exercise: 11

**Find the following integrals:**

- $\int \frac{x+3}{\sqrt{5-4x+x^2}} dx$
- $\int \frac{4x+1}{\sqrt{2x^2+x-3}} dx$
- $\int \sqrt{\frac{a-x}{a+x}} dx$

- $\int x \sqrt{\frac{a^2-x^2}{a^2+x^2}} dx$
- $\int \sqrt{\frac{1-x}{1+x}} dx$

**Integral of the form:**

$$\int \frac{1}{a \sin^2 x + b \cos^2 x} dx, \int \frac{1}{a + b \sin^2 x} dx, \int \frac{1}{a + b \cos^2 x} dx, \int \frac{1}{(a \sin x + b \cos x)^2} dx, \int \frac{1}{a + b \sin^2 x + c \cos^2 x} dx$$

To evaluate this type of integral use the following steps:

- Divide numerator and denominator by  $\cos^2 x$ .
- Replace  $\sec^2 x$ , if any in the denominator by  $(1 + \tan^2 x)$ .
- Put  $\tan x = t$ , then we get integral of the form  $\int \frac{1}{at^2 + bt + c} dt$

**Example Exercise: 12**

- $\int \frac{\sin x}{\sin 3x} dx$
- $\int \frac{1}{(\sin x - 2 \cos x)(2 \sin x + \cos x)} dx$

**Answers**

- $\frac{1}{2\sqrt{3}} \log \left| \frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x} \right| + C$
- $\frac{1}{5} \log \left| \frac{\tan x - 2}{2 \tan x + 1} \right| + C$

**Practice Exercise: 12**

**Find the following integrals:**

- $\int \frac{1}{1 + 3 \sin^2 x + 8 \cos^2 x} dx$
- $\int \frac{1}{1 + 3 \sin^2 x} dx$
- $\int \frac{1}{3 + 2 \cos^2 x} dx$
- $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$
- $\int \frac{1}{\cos 2x + 3 \sin^2 x} dx$

**Integral of the form:**

$$\int \frac{1}{a \sin x + b \cos x} dx, \int \frac{1}{a + b \sin x} dx, \int \frac{1}{a + b \cos x} dx, \int \frac{1}{a \sin x + b \cos x + c} dx,$$

To evaluate this type of integral use the following steps:

(i) Put  $\sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$ ,  $\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

(ii) Replace  $1 + \tan^2 \frac{x}{2}$  in the numerator by  $\sec^2 \frac{x}{2}$

(iii) Put  $\tan \frac{x}{2} = t$ , we get integral of the form :  $\int \frac{1}{at^2 + bt + c} dt$

### Example Exercise: 13

1.  $\int \frac{1}{2 + \cos x} dx$

2.  $\int \frac{1}{3 + 2 \sin x + \cos x} dx$

### Answers

1.  $\frac{2}{\sqrt{3}} \tan^{-1} \left( \frac{\tan \frac{x}{2}}{\sqrt{3}} \right) + C$

2.  $\tan^{-1} (1 + \tan \frac{x}{2}) + C$

### Practice Exercise: 13

Find the following integrals:

1.  $\int \frac{1}{5 - 4 \sin x} dx$

2.  $\int \frac{1}{4 \cos x - 1} dx$

3.  $\int \frac{1}{\sqrt{3} \sin x + \cos x} dx$

**Integration By Parts:**  $\int_I f(x) \cdot \int_{II} g(x) dx = f(x) \int g(x) dx - \int \left\{ \frac{d}{dx} f(x) \cdot \int g(x) dx \right\} dx$

We can choose first function as the function which comes first in the word “ILATE” where



**I** – stands for the inverse trigonometric functions.

**L** – stands for the logarithmic functions.

**A** – stands for the algebraic functions.

**T** – stands for the trigonometric functions.

**E** – stands for the exponential functions.

### Example Exercise: 14

1.  $\int x^3 \log 2x dx$

2.  $\int x^2 \tan^{-1} x dx$

3.  $\int \sin^3 \sqrt{x} dx$

4.  $\int \log(1 + x^2) dx$

### Answers

1.  $\frac{x^4}{4} \log 2x - \frac{1}{16} x^4 + C$

2.  $\frac{x^3}{3} \tan^{-1} x - \frac{1}{6} x^2 + \frac{1}{6} \log |x^2 + 1| + C$

3.  $-3x^{3/2} \cos^3 \sqrt{x} + 6x^{1/3} \sin^3 \sqrt{x} + 6 \cos^3 \sqrt{x} + C$

4.  $x \log(x^2 + 1) - 2x + 2 \tan^{-1} x + C$

### Practice Exercise: 14

Find the following integrals:

1.  $\int x \cos x dx$

2.  $\int \log x dx$

3.  $\int xe^x dx$

4.  $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$

5.  $\int x \sin x dx$

6.  $\int x \sin 3x dx$

7.  $\int x^2 e^x dx$

8.  $\int x \log x dx$

9.  $\int x \log 2x dx$

10.  $\int x^2 \log x dx$

11.  $\int x \sin^{-1} x dx$

12.  $\int (\sin^{-1} x)^2 dx$

13.  $\int \tan^{-1} x dx$

14.  $\int x(\log x)^2 dx$

15.  $\int (x^2 + 1) \log x dx$

16.  $\int x \sin^2 x dx$

17.  $\int (\log x)^2 dx$

18.  $\int \frac{\log x}{x^2} dx$

19.  $\int \log(1+x^2) dx$

20.  $\int \sec^3 x dx$

21.  $\int \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx$

22.  $\int \frac{\sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x}}{\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}} dx$

23.  $\int \frac{\log(\log x)}{x} dx$

24.  $\int \frac{\log x}{x^n} dx$

25.  $\int 2x^3 e^{x^2} dx$

$\int e^{\sqrt{x}} dx$

26.  $\int \log_{10} x dx$

27.  $\int \sin^3 \sqrt{x} dx$

28.  $\int \cos ec^3 x dx$

29.  $\int x \left( \frac{\sec 2x - 1}{\sec 2x + 1} \right) dx$

30.  $\int (x+1)e^x \log(xe^x) dx$

31.  $\int \tan^{-1} \left( \frac{3x-x^3}{1-3x^2} \right) dx$

32.  $\int (e^{\log x} + \sin x) \cos x dx$

**Integral of the form:**  $\int e^x \{f(x) + f'(x)\} dx$ , to evaluate this type of integral use the following steps:

(i) Write  $\int e^x \{f(x) + f'(x)\} dx = \int e^x f(x) dx + \int e^x f'(x) dx$

(ii) Evaluate first integral on R.H.S. by using integration by parts. Then we get

$$\int_{II} e^x f(x) dx + \int_I e^x f'(x) dx = e^x f(x) - \int e^x f'(x) dx + \int e^x f'(x) dx$$

$$\Rightarrow \int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$$

**Example Exercise: 15**

1.  $\int e^x \frac{x^2 + 1}{(x+1)^2} dx$

2.  $\int \frac{e^x}{x} [x(\log x)^2 + 2 \log x] dx$

3.  $\int \frac{\sqrt{1-\sin x}}{1+\cos x} e^{-x/2} dx$

**Answers**

1.  $e^x - \frac{2e^x}{x+1} + C$

2.  $e^x (\log x)^2 + C$

3.  $-e^{-x/2} \sec\left(\frac{x}{2}\right) + C$

**Practice Exercise: 15**

Find the following integrals:

1.  $\int e^x \left( \tan^{-1} x + \frac{1}{1+x^2} \right) dx$

$$2. \int \frac{(x^2+1)e^x}{(x+1)^2} dx$$

$$3. \int e^x (\sin x + \cos x) dx$$

$$4. \int \frac{xe^x}{(1+x)^2} dx$$

$$5. \int e^x \left( \frac{1+\sin x}{1+\cos x} \right) dx$$

$$6. \int e^x \left( \frac{1}{x} - \frac{1}{x^2} \right) dx$$

$$7. \int \frac{(x-3)e^x}{(x-1)^3} dx$$

$$8. \int e^x \frac{x}{(x+1)^2} dx$$

$$9. \int e^x \left( \frac{2+\sin 2x}{1+\cos 2x} \right) dx$$

$$10. \int \{ \sin(\log x) + \cos(\log x) \} dx$$

$$11. \int \frac{\log x}{(1+\log x)^2} dx$$

$$12. \int e^{2x} \left( \frac{1+\sin 2x}{1+\cos 2x} \right) dx$$

$$13. \int e^x \frac{\sqrt{1-x^2} \sin^{-1} x + 1}{\sqrt{1-x^2}} dx$$

$$14. \int e^x \left( \log x + \frac{1}{x^2} \right) dx$$

$$15. \int \left( \frac{1}{\log x} - \frac{1}{(\log x)^2} \right) dx$$

### Integration of rational algebraic functions by using Partial fractions

If  $f(x)$  and  $g(x)$  are two polynomial then  $\frac{f(x)}{g(x)}$  is defined as rational algebraic function or rational function

#### Note:

- (i) If degree of  $f(x) <$  degree of  $g(x)$ , then  $\frac{f(x)}{g(x)}$  are proper rational functions
- (ii) If degree of  $f(x) \geq$  degree of  $g(x)$ , then  $\frac{f(x)}{g(x)}$  are improper rational functions so divide

$f(x)$  by  $g(x)$  to represent it in proper rational expression ( $\therefore$  Method of partial fraction is applicable for proper rational functions)

#### Forms of Partial fractions:

❖ When denominator has non repeated linear factor:

$$i. \frac{px+q}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$$

$$ii. \frac{px^2+qx+r}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$$

❖ When denominator has repeated linear factor:

$$iii. \frac{px+q}{(x-a)^2} = \frac{A}{x-a} + \frac{B}{(x-a)^2}$$

$$iv. \frac{px^2+qx+r}{(x-a)^2(x-b)} = \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$$

❖ When denominator has non repeated quadratic factor:

$$v. \frac{px^2+qx+r}{(x-a)(ax^2+bx+c)} = \frac{A}{x-a} + \frac{Bx+C}{ax^2+bx+c}$$

Find the values of A, B, C by comparing the coefficients and taking integration on both sides after putting these values.

**Example Exercise: 16**

1.  $\int \frac{2x-1}{(x-1)(x+2)(x-3)} dx$
2.  $\int \frac{x^3}{(x-1)(x-2)} dx$
3.  $\int \frac{x^2+1}{(x-1)^2(x+3)} dx$
4.  $\int \frac{x}{(x-1)(x^2+4)} dx$
5.  $\int \frac{\sin x}{\sin 4x} dx$

**Answers**

1.  $-\frac{1}{6} \log|x-1| - \frac{1}{3} \log|x+2| + \frac{1}{2} \log|x-3| + C$
2.  $\frac{x^2}{2} + 3x - \log|x-1| + 8 \log|x-2| + C$
3.  $\frac{3}{8} \log|x-1| - \frac{1}{2(x-1)} + \frac{5}{8} \log|x+3| + C$
4.  $\frac{1}{5} \log|x-1| - \frac{1}{10} \log(x^2+4) + \frac{2}{5} \tan^{-1}\left(\frac{x}{2}\right) + C$
5.  $-\frac{1}{8} \log \left| \frac{1+\sin x}{1-\sin x} \right| + \frac{1}{4\sqrt{2}} \log \left| \frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x} \right| + C$

**Practice Exercise: 16**

Find the following integrals:

1.  $\int \frac{3x-2}{(x+1)^2(x+3)} dx$
2.  $\int \frac{dx}{(x+1)(x+2)}$

3.  $\int \frac{x^2+1}{x^2-5x+6} dx$
4.  $\int \frac{x^2}{(x^2+1)(x^2+4)} dx$
5.  $\int \frac{(3\sin\phi-2)\cos\phi}{5-\cos^2\phi-4\sin\phi} d\phi$
6.  $\int \frac{x^2+x+1}{(x+2)(x^2+1)} dx$
7.  $\int \frac{1}{x^4-1} dx$
8.  $\int \frac{1}{x(x^n+1)} dx$
9.  $\int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx$
10.  $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx$
11.  $\int \frac{(1-\cos x)}{\cos x(1+\cos x)} dx$
12.  $\int \frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)} dx$
13.  $\int \frac{\tan\theta + \tan^3\theta}{1+\tan^3\theta} d\theta$
14.  $\int \frac{1}{x \log x (2+\log x)} dx$
15.  $\int \frac{1}{x[6(\log x)^2 + 7\log x + 2]} dx$

**Some more important forms**

**Integral of the form:**  $\int \frac{x^2+1}{x^4+Ax^2+1} dx$ ,  $\int \frac{x^2-1}{x^4+Ax^2+1} dx$ ,  $\int \frac{1}{x^4+Ax^2+1} dx$ , Where  $A \in \mathbb{R}$ ,

To evaluate this type of integrals use the following steps:

- (i) Divide numerator and denominator by  $x^2$
- (ii) Express denominator in the form of  $\left(x + \frac{1}{x}\right)^2$  if numerator is  $\left(1 - \frac{1}{x^2}\right)$  or  $\left(x - \frac{1}{x}\right)^2$  if numerator is  $\left(1 + \frac{1}{x^2}\right)$
- (iii) Put  $\left(x + \frac{1}{x}\right) = t$  or  $\left(x - \frac{1}{x}\right) = t$ , then we get integral of the form  $\int \frac{1}{x^2+a^2} dx$ , or  $\int \frac{1}{x^2-a^2} dx$

### Example Exercise: 17

1.  $\int \frac{x^2-1}{x^4+x^2+1} dx$
2.  $\int \frac{x^2}{x^4+1} dx$
3.  $\int \sqrt{\tan x} dx$
4.  $\int \frac{1}{x^4+1} dx$

#### Answers

1.  $\frac{1}{2} \log \left| \frac{x^2-x+1}{x^2+x+1} \right| + C$
2.  $\frac{1}{2\sqrt{2}} \tan^{-1} \left( \frac{x^2-1}{x\sqrt{2}} \right) + \frac{1}{4\sqrt{2}} \log \left| \frac{x^2-x\sqrt{2}+1}{x^2+x\sqrt{2}+1} \right| + C$

3.  $\frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{\tan x - 1}{\sqrt{2} \tan x} \right) + \frac{1}{2\sqrt{2}} \log \left| \frac{\tan x - \sqrt{2} \tan x + 1}{\tan x + \sqrt{2} \tan x + 1} \right| + C$
4.  $\frac{1}{2\sqrt{2}} \tan^{-1} \left( \frac{x^2-1}{\sqrt{2}x} \right) - \frac{1}{4\sqrt{2}} \log \left| \frac{x^2 - \sqrt{2}x + 1}{x^2 + \sqrt{2}x + 1} \right| + C$

### Practice Exercise: 17

Find the following integrals:

1.  $\int \frac{x^2+1}{x^4+1} dx$
2.  $\int \frac{1}{\sin^4 x + \cos^4 x} dx$
3.  $\int (\sqrt{\tan \theta} + \sqrt{\cot \theta}) d\theta$
4.  $\int (\sqrt{\cot \theta}) d\theta$
5.  $\int \frac{(x-1)^2}{x^4+x^2+1} dx$

**Integral of the form:**  $\int \frac{f(x)}{P\sqrt{Q}} dx$ , Where P and Q are polynomial functions in x.

- ❖  $\int \frac{1}{(ax+b)\sqrt{cx+d}} dx$ , To evaluate put  $cx+d = t^2$
- ❖  $\int \frac{1}{(ax^2+bx+c)\sqrt{px+q}} dx$ , To evaluate put  $px+q = t^2$
- ❖  $\int \frac{1}{(ax+b)\sqrt{px^2+qx+r}} dx$ , To evaluate put  $ax+b = \frac{1}{t}$
- ❖  $\int \frac{1}{(ax^2+b)\sqrt{px^2+q}} dx$ , To evaluate put  $x = \frac{1}{t}$

### Example Exercise: 18

- $\int \frac{x^2}{(x-1)\sqrt{x+2}} dx$
- $\int \frac{x}{(x^2+2x+2)\sqrt{x+1}} dx$
- $\int \frac{1}{(x+1)\sqrt{x^2+x+1}} dx$
- $\int \frac{x}{(x^2+4)\sqrt{x^2+9}} dx$

**Answers**

- $\frac{2}{3}(x+2)^{3/2} - 2\sqrt{x+2} + \frac{1}{\sqrt{3}} \log \left| \frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}} \right| + C$
- $\frac{1}{\sqrt{2}} \log \left| \frac{(x+2)-\sqrt{2(x+1)}}{(x+2)+\sqrt{2(x+1)}} \right| + C$

- $-\log \left| \frac{1}{x+1} - \frac{1}{2} + \frac{\sqrt{x^2+x+1}}{x+1} \right| + C$
- $\frac{1}{2\sqrt{5}} \log \left| \frac{\sqrt{x^2+9} - \sqrt{5}}{\sqrt{x^2+9} + \sqrt{5}} \right| + C$

**Practice Exercise: 18**

Find the following integrals:

- $\int \frac{1}{(x^2-4)\sqrt{x+1}} dx$
- $\int \frac{1}{(x-1)\sqrt{x^2+4}} dx$
- $\int \frac{\sqrt{1+x^2}}{1-x^2} dx$

**Integral of the form:**  $\int \sqrt{ax^2+bx+c} dx$ , to evaluate use the following steps:

- Make coefficient of  $x^2$  unity, if it is not, by multiplying and dividing by it.
- Add and subtract the square of half of the coefficient of  $x$  to the express  $ax^2 +$

$$bx + c \text{ in the form of } \left[ \left( x + \frac{b}{2a} \right)^2 + \frac{4ac - b^2}{4a^2} \right] \text{ or } \left[ \frac{4ac - b^2}{4a^2} - \left( x + \frac{b}{2a} \right)^2 \right]$$

- Use the suitable formula from the following formulas:

- $\int \sqrt{a^2 - x^2} dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{1}{2} a^2 \sin^{-1} \left( \frac{x}{a} \right) + C$
- $\int \sqrt{a^2 + x^2} dx = \frac{1}{2} x \sqrt{a^2 + x^2} + \frac{1}{2} a^2 \log \left| x + \sqrt{a^2 + x^2} \right| + C$
- $\int \sqrt{x^2 - a^2} dx = \frac{1}{2} x \sqrt{x^2 - a^2} - \frac{1}{2} a^2 \log \left| x + \sqrt{x^2 - a^2} \right| + C$

**Example Exercise: 19**

- $\int \frac{\sqrt{16 + (\log x)^2}}{x} dx$
- $\int \sqrt{1+x-2x^2} dx$

**Answers**

- $\frac{1}{2} \log x \sqrt{(\log x)^2 + 16} + 8 \log \left| \log x + \sqrt{(\log x)^2 + 16} \right| + C$
- $\frac{1}{8} (4x-1) \sqrt{1+x-2x^2} + \frac{9\sqrt{2}}{32} \sin^{-1} \left( \frac{4x-1}{3} \right) + C$



**Integral of the form:**  $\int (px+q)\sqrt{ax^2+bx+c} dx$ , To evaluate use the following steps:

- (i) Let  $px+q = A \frac{d}{dx}(ax^2+bx+c) + B$   
 i.e.  $px+q = A(2ax+b) + B \dots \dots (1)$
- (ii) Obtain the values of A and B by equating the coefficient of like powers of x on both sides
- (iii) Put the values of A and B in equation (1) multiplying by  $\sqrt{ax^2+bx+c}$  on both sides of equation (1) and taking integration on both the sides. then we get
- $$\int (px+q)\sqrt{ax^2+bx+c} dx = A \int (2ax+b)\sqrt{ax^2+bx+c} dx + B \int \sqrt{ax^2+bx+c} dx$$
- (iv) Integrate R.H.S. of step (iii)

**Example Exercise: 20**

1.  $\int (2x+3)\sqrt{x^2+4x+3} dx$

1.  $\frac{2}{3}(x^2+4x+3)^{3/2} - \left[ \frac{1}{2}(x+2)\sqrt{x^2+4x+3} - \frac{1}{2} \log \left| (x+2) + \sqrt{x^2+4x+3} \right| \right] + C$

2.  $\frac{1}{3}(x+x^2)^{3/2} - \frac{1}{8}(2x+1)\sqrt{x^2+x} + \frac{1}{16} \log \left| \left( x + \frac{1}{2} \right) + \sqrt{x^2+x} \right| + C$

2.  $\int x\sqrt{x^2+x} dx$

**Answers**

**Integral of the form:**  $\int \frac{asinx+bcosx}{csinx+dcosx} dx$ , to evaluate use the following steps:

- (i) Write:  $asinx+bcosx = A \frac{d}{dx}(csinx+dcosx) + B(csinx+dcosx)$
- (ii) Obtain the values of A and B by equating the coefficient of cosx and sinx, then we get

$$\int \frac{asinx+bcosx}{csinx+dcosx} dx = A \int \frac{ccosx-dsinx}{csinx+dcosx} dx + B \int \frac{csinx+dcosx}{csinx+dcosx} dx$$

Integrate R.H.S. of step (ii) by using suitable procedure.

**Example Exercise: 21**

1.  $\int \frac{3sinx+2cosx}{3cosx+2sinx} dx$

$\left( \text{Ans: } -\frac{5}{13}x + \frac{12}{13} \log |3cosx+2sinx| + C \right)$

**Integral of the form:**  $\int e^{ax}sinbx dx$ ,  $\int e^{ax}cosbx dx$ , To evaluate this type of integral take  $e^{ax}$  as a first function and cosbx or sinbx as second function then expand integral by using integration by parts.

**Example Exercise: 22**

1.  $\int e^{2x}sin3x dx$   $\left[ \text{Ans: } \frac{e^{2x}}{13}(2sin3x-3cos3x) + C \right]$

**QUESTION BANK**

1.  $\int \frac{(2^x + 3^x)^2}{6^x} dx$
2.  $\int \cos ec^2 x \cos^2 2x dx$
3.  $\int \frac{\cos^7 x}{\sin x} dx$
4.  $\int \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$
5.  $\int \frac{\sin x - \cos x}{\sqrt{\sin 2x}} dx$
6.  $\int \frac{1}{\sin(x-a)\sin(x-b)} dx$
7.  $\int \frac{1}{\cos(x-a)\cos(x-b)} dx$
8.  $\int \frac{1}{\sin(x-a)\cos(x-b)} dx$
9.  $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$
10.  $\int \sqrt{\frac{1+x}{x}} dx$
11.  $\int \sqrt{\frac{1-x}{x}} dx$
12.  $\int \frac{\sqrt{a-x} - \sqrt{x}}{1-\sqrt{ax}} dx$
13.  $\int \frac{1}{a+b \tan x} dx$
14.  $\int \sec^4 x dx$
15.  $\int \sec^6 x dx$
16.  $\int \frac{1}{x\sqrt{1+x^n}} dx$
17.  $\int \tan^{-1} \sqrt{\frac{1-x}{1+x}} dx$
18.  $\int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$
19.  $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$
20.  $\int \frac{\sin x + \cos x}{9+16 \sin 2x} dx$
21.  $\int \left\{ \log(\log x) + \frac{1}{(\log x)^2} \right\} dx$

22.  $\int \sqrt{\frac{\sin(x-\alpha)}{\sin(x+\alpha)}} dx$
23.  $\int \frac{\sqrt{\cos 2x}}{\sin x} dx$
24.  $\int \frac{(x^4 - x)^{\frac{1}{4}}}{x^5} dx$
25.  $\int \frac{1}{x^{\frac{1}{2}} + x^{\frac{1}{3}}} dx$
26.  $\int \frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}} dx$
27.  $\int \frac{\sqrt{x^2+1} [\log(x^2+1) - 2 \log x]}{x^4} dx$
28.  $\int \frac{1}{x\sqrt{ax-x^2}} dx$
29.  $\int |x| dx$
30.  $\int \sin^2 \left( 2 \tan^{-1} \sqrt{\frac{1+x}{1-x}} \right) dx$
31.  $\int 2^{2^{2^x}} 2^{2^x} 2^x dx$
32.  $\int \sqrt{\frac{x}{a^3-x^3}} dx$
33.  $\int \sin^{-1} \left( \frac{6x}{1+9x^2} \right) dx$
34.  $\int [1 + 2 \tan x (\tan x + \sec x)]^{\frac{1}{2}} dx$
35.  $\int \frac{\sin(x+a)}{\sin(x+b)} dx$
36.  $\int \tan x \tan 2x \tan 3x dx$
37.  $\int \sec^n x \tan x dx$
38.  $\int 5^{x+\tan^{-1} x} \left( \frac{x^2+2}{x^2+1} \right) dx$
39.  $\int \frac{1}{x^2 (x^4+1)^{\frac{3}{4}}} dx$
40.  $\int \sin^{-1} \sqrt{x} dx$

## ANSWERS

### Practice Exercise: 1

1.  $\frac{x^2}{2} + \frac{1}{x} 3 + C$
2.  $\frac{3}{5} x^{\frac{5}{3}} + C$
3.  $\frac{2}{5} x^{\frac{5}{2}} + 2e^x - \log|x| + C$
4.  $-\cos x + \sin x + C$
5.  $-\cot x - \cos ecx + C$
6.  $\tan x - \sec x + C$
7.  $\frac{x^3}{3} - x + C$
8.  $\frac{ax^3}{3} + \frac{bx^2}{2} + cx + C$
9.  $\frac{2}{3} x^3 + e^x + C$
10.  $\frac{x^2}{2} + \log|x| - 2x + C$
11.  $\frac{x^2}{2} + 5x + \frac{4}{x} + C$
12.  $\frac{2}{7} x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8\sqrt{x} + C$
13.  $\frac{x^3}{3} + x + C$
14.  $\frac{2}{3} x^{\frac{3}{2}} - \frac{2}{5} x^{\frac{5}{2}} + C$
15.  $x^2 - 3\sin x + e^x + C$
16.  $\frac{2}{3} x^3 + 3\cos x + \frac{10}{3} x^{\frac{3}{2}} + C$
17.  $\tan x + \sec x + C$
18.  $\tan x - x + C$
19.  $2\tan x - 3\sec x + C$
20.  $\tan x - \cot x + C$
21.  $-\sqrt{2}\cos x + C$
22.  $-\cos x + \sin x + C$
23.  $\tan x - \sec x + C$
24.  $\frac{\pi}{2} x - \frac{x^2}{2} + C$
25.  $\frac{x^2}{2} + C$
26.  $\frac{x^3}{3} - x + \tan^{-1} x + C$

$$27. \frac{\left(\frac{a}{b}\right)^x}{\log_e\left(\frac{a}{b}\right)} + \frac{\left(\frac{b}{a}\right)^x}{\log_e\left(\frac{b}{a}\right)} + 2x + C$$

$$28. \frac{x^3}{3} + C$$

$$29. -\cos(e^x) + C$$

$$30. \frac{a^x e^x}{\log(ae)} + C$$

$$31. e^x + \tan^{-1} x$$

$$32. f(x) = 2\cos x + 4\sin x + 1$$

### Practice Exercise: 2

1.  $\log|x+1| - \frac{1}{|x+1|} + C$
2.  $\frac{1}{2} e^{2x-3} + C$
3.  $\frac{2}{27} \left\{ (3x+4)^{\frac{3}{2}} + (3x+1)^{\frac{3}{2}} \right\} + C$
4.  $\frac{1}{3} (4x+7)^{\frac{3}{2}} - \frac{1}{2} (4x+7)^{\frac{1}{2}} + C$
5.  $\frac{14}{45} (3x+2)^{\frac{5}{2}} - \frac{40}{27} (3x+2)^{\frac{3}{2}} + C$
6.  $\frac{x^3}{3} - x^2 + 4x - 8\log|x+2| + C$
7.  $\frac{1}{2} e^{2x} + 2x - \frac{1}{2} e^{-2x} + C$
8.  $\frac{1}{3} (e^x + 1)^3 + C$
9.  $-\frac{1}{4a} \cos 2(ax+b) + C$

### Practice Exercise: 3

1.  $\frac{1}{2} \left( x - \frac{\sin 2x}{2} \right) + C$
2.  $\frac{1}{2} \left( x + \frac{\sin 2x}{2} \right) + C$
3.  $\frac{1}{4} \left[ \frac{\sin 3x}{3} + 3\sin x \right] + C$
4.  $\frac{1}{8} \left[ 3x + 2\sin 2x + \frac{\sin 4x}{4} \right] + C$
5.  $\frac{1}{128} \left[ 3x - \sin 4x + \frac{1}{8} \sin 8x \right] + C$

$$6. \frac{3x}{8} + \frac{\sin 4x}{8} + \frac{\sin 8x}{64} + C$$

$$7. \frac{x}{2} - \frac{\sin 2bx}{4b} + C$$

$$8. \frac{1}{2}(x - \sin x) + C$$

### Practice Exercise: 4

$$1. \frac{1}{2} \left[ -\frac{\cos 7x}{7} + \cos x \right] + C$$

$$2. \frac{1}{4} \left[ x + \frac{\sin 12x}{12} + \frac{\sin 8x}{8} + \frac{\sin 4x}{4} \right] + C$$

$$3. -\cos 2x + C$$

$$4. 2 \left[ \frac{\sin 3x}{3} + \sin x \right] + C$$

$$5. 2 \left[ \sin \frac{x}{2} \pm \cos \frac{x}{2} \right] + C$$

$$6. \frac{1}{2} \left[ -\frac{\cos(m+n)x}{m+n} - \frac{\cos(m-n)x}{m-n} \right] + C$$

### Practice Exercise: 5

$$1. -\cos(x^2 + 1) + C$$

$$2. \frac{2}{5} \tan^5 \sqrt{x} + C$$

$$3. -\cos(\tan^{-1} x) + C$$

$$4. x \cos a - \sin a \log |\sin(x+a)| + C$$

$$5. \frac{x}{2} + \frac{1}{2} \log |\cos x + \sin x| + C$$

$$6. \log(1+x^2) + C$$

$$7. \log|1 + \log x| + C$$

$$8. \cos(\cos x) + C$$

$$9. \frac{1}{6}(1+2x^2)^{3/2} + C$$

$$10. \frac{4}{3}(x^2+x+1)^{3/2} + C$$

$$11. 2 \log |\sqrt{x}-1| + C$$

$$12. -\frac{1}{8} \log |9-4x^2| + C$$

$$13. -\frac{1}{2e^{x^2}} + C$$

$$14. e^{\tan^{-1} x} + C$$

$$15. \log |e^x + e^{-x}| + C$$

$$16. \frac{1}{2} \log(e^{2x} + e^{-2x}) + C$$

$$17. \frac{1}{2} (\sin^{-1} x)^2 + C$$

$$18. \frac{1}{2} \log |2 \sin x + 3 \cos x| + C$$

$$19. \frac{1}{1 - \tan x} + C$$

$$20. 2 \sin \sqrt{x} + C$$

$$21. 2\sqrt{1 + \sin x} + C$$

$$22. \frac{1}{2} (\log \sin x)^2 + C$$

$$23. -\log |1 + \cos x| + C$$

$$24. \frac{x}{2} - \frac{1}{2} \log |\cos x + \sin x| + C$$

$$25. \frac{x}{2} - \frac{1}{2} \log |\cos x - \sin x| + C$$

$$26. 2\sqrt{\tan x} + C$$

$$27. -\frac{1}{4} \cos(\tan^{-1} x^4) + C$$

$$28. \log(10^x + x^{10}) + C$$

$$29. \log |1 + e^x| + C$$

$$30. \frac{1}{(a^2 - b^2)} \log |a^2 \sin^2 x + b^2 \cos^2 x| + C$$

$$31. \log(\log \tan x) + C$$

$$32. -\frac{a}{b} \log |be^{-x} + c| + C$$

$$33. -\log |1 + e^{-x}| + C$$

$$34. \log \left| \log \tan \frac{x}{2} \right| + C$$

$$35. \log |\log(\sec x + \tan x)| + C$$

$$36. \log \{ \log(\log x) \} + C$$

$$37. \frac{1}{e} \log |e^x + x^e| + C$$

$$38. \log |x + \log x| + C$$

### Practice Exercise: 6

$$1. \frac{1}{3} (\log |x|)^3 + C$$

$$2. \frac{1}{7} (x^3 - 1)^{7/3} + \frac{1}{4} (x^3 - 1)^{4/3} + C$$

$$3. -\frac{1}{18(2+3x^3)^2} + C$$

4.  $\frac{(\log x)^{1-m}}{1-m} + C$
5.  $\frac{1}{3}(\sin 2x)^{3/2} + C$
6.  $\frac{1}{1+\cos x} + C$
7.  $\frac{1}{3}(1+\log x)^3 + C$
8.  $\frac{1}{3}(x+\log x)^3 + C$
9.  $(\sin^{-1} x)^4 + C$
10.  $-\frac{1}{1+e^x} + C$
11.  $\frac{\{e^{\sin^{-1} x}\}^2}{2} + C$

### Practice Exercise: 7

1.  $-\left[\cos x - \frac{2}{3}\cos^3 x + \frac{1}{5}\cos^5 x\right] + C$
2.  $\frac{1}{5}\sin^5 x - \frac{\sin^7 x}{7} + C$
3.  $-\cos x + \cos^3 x - \frac{3}{5}\cos^5 x + \frac{1}{7}\cos^7 x + C$
4.  $-\frac{1}{6}\cos^6 x + \frac{1}{8}\cos^8 x + C$

### Practice Exercise: 8

1.  $\frac{1}{8}\log\left|\frac{x-4}{x+4}\right| + C$
2.  $\frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right) + C$
3.  $\frac{1}{17}\log\left|\frac{3x-2}{x+5}\right| + C$
4.  $\tan^{-1} x^3 + C$
5.  $\frac{3}{2\sqrt{2}}\tan^{-1}\sqrt{2}x^2 + C$
6.  $\frac{1}{6}\log\left|\frac{1+x^3}{1-x^3}\right| + C$
7.  $\frac{1}{6}\tan^{-1}\left(\frac{3x+1}{2}\right) + C$
8.  $\tan^{-1}(x+1) + C$

### Practice Exercise: 9

1.  $\sin^{-1}(x-1) + C$
2.  $\frac{1}{\sqrt{5}}\log\left|x - \frac{1}{5} + \sqrt{x^2 - \frac{2x}{5}}\right| + C$
3.  $\frac{1}{2}\log\left|2x + \sqrt{1+4x^2}\right| + C$
4.  $\log\left|\frac{1}{2-x+\sqrt{x^2-4x+5}}\right| + C$
5.  $\sqrt{x^2-1} - \log\left|x + \sqrt{x^2-1}\right| + C$
6.  $\frac{1}{3}\log\left|x^3 + \sqrt{x^6+a^6}\right| + C$
7.  $\log\left|\tan x + \sqrt{\tan^2 x + 4}\right| + C$
8.  $\log\left|x+1 + \sqrt{x^2+2x+2}\right| + C$
9.  $\log\left|x - \frac{3}{2} + \sqrt{x^2-3x+2}\right| + C$
10.  $\log\left|x - \frac{a+b}{2} + \sqrt{(x-a)(x-b)}\right| + C$

### Practice Exercise: 10

1.  $\frac{1}{4}\log|2x^2+6x+5| + \frac{1}{2}\tan^{-1}(2x+3) + C$
2.  $\frac{1}{2}\log|x^2-2x-5| + \frac{2}{\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C$
3.  $\frac{5}{6}\log|3x^2+2x+1| - \frac{11}{3\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) + C$
4.  $\frac{1}{4}\log|x^4-9| + \frac{1}{12}\log\left|\frac{x^2-3}{x^2+3}\right| + C$
5.  $-\frac{1}{2}\log|e^{-2x}+3e^{-x}+2| + \frac{3}{2}\log\left|\frac{e^{-x}+1}{e^{-x}+2}\right| + C$
6.  $3\log|2-\sin x| + \frac{4}{2-\sin x} + C$

### Practice Exercise: 11

1.  $-\sqrt{5-4x-x^2} + \sin^{-1}\left(\frac{x+2}{3}\right) + C$
2.  $2\sqrt{2x^2+x-3} + C$
3.  $a\sin^{-1}\left(\frac{x}{a}\right) + \sqrt{a^2-x^2} + C$
4.  $\frac{1}{2}a^2\sin^{-1}\left(\frac{x^2}{a^2}\right) + \frac{1}{2}\sqrt{a^4-x^4} + C$

5.  $\sin^{-1} x + \sqrt{1-x^2} + C$

**Practice Exercise: 12**

1.  $\frac{1}{6} \tan^{-1} \left( \frac{2 \tan x}{3} \right) + C$

2.  $\frac{1}{2} \tan^{-1} (2 \tan x) + C$

3.  $\frac{1}{\sqrt{15}} \tan^{-1} \left( \frac{\sqrt{3} \tan x}{\sqrt{5}} \right) + C$

4.  $\tan^{-1} (\tan^2 x) + C$

5.  $\frac{1}{\sqrt{2}} \tan^{-1} (\sqrt{2} \tan x) + C$

**Practice Exercise: 13**

1.  $\frac{2}{3} \tan^{-1} \left( \frac{5 \tan \left( \frac{x}{2} \right) - 4}{3} \right) + C$

2.  $\frac{1}{\sqrt{15}} \log \left| \frac{\sqrt{3} + \sqrt{5} \tan \frac{x}{2}}{\sqrt{3} - \sqrt{5} \tan \frac{x}{2}} \right| + C$

3.  $\frac{1}{2} \log \left| \tan \left( \frac{x}{2} + \frac{\pi}{12} \right) \right| + C$

**Practice Exercise: 14**

1.  $x \sin x + \cos x + C$

2.  $x \log x - x + C$

3.  $xe^x - e^x + C$

4.  $x - \sqrt{1-x^2} \sin^{-1} x + C$

5.  $-x \cos x + \sin x + C$

6.  $-\frac{x}{3} \cos 3x + \frac{1}{9} \sin 3x + C$

7.  $e^x (x^2 - 2x + 2) + C$

8.  $\frac{x^2}{2} \log x - \frac{x^2}{4} + C$

9.  $\frac{x^2}{2} \log 2x - \frac{x^2}{4} + C$

10.  $\frac{x^3}{3} \log x - \frac{x^3}{9} + C$

11.  $\frac{1}{4} (2x^2 - 1) \sin^{-1} x + \frac{x\sqrt{1-x^2}}{4} + C$

12.  $(\sin^{-1} x)^2 x + 2\sqrt{1-x^2} \sin^{-1} x - 2x + C$

13.  $x \tan^{-1} x - \frac{1}{2} \log (1+x^2) + C$

14.  $\frac{x^2}{2} (\log x)^2 - \frac{x^2}{2} \log x + \frac{x^2}{4} + C$

15.  $\left( \frac{x^3}{3} + x \right) \log x - \frac{x^3}{9} - x + C$

16.  $\frac{1}{9} x^2 - \frac{x}{4} \sin 2x - \frac{1}{8} \cos 2x + C$

17.  $x (\log x)^2 - 2[x \log x - x] + C$

18.  $-\frac{1}{x} (1 + \log x) + C$

19.  $x \log (x^2 + 1) - 2x + 2 \tan^{-1} x + C$

20.  $\frac{1}{2} \sec x \tan x + \frac{1}{2} \log |\sec x + \tan x| + C$

21.  $\frac{x}{\sqrt{1-x^2}} \sin^{-1} x + \frac{1}{2} \log |1-x^2| + C$

22.  $\frac{2}{\pi} \left[ \sqrt{x-x^2} - (1-2x) \sin^{-1} \sqrt{x} \right] - x + C$

23.  $\log x [\log (\log x) - 1] + C$

24.  $\frac{x^{1-n}}{1-n} \log x - \frac{x^{1-n}}{(1-n)^2} + C$

25.  $e^{x^2} (x^2 - 1) + C$

26.  $2e^{\sqrt{x}} (\sqrt{x} - 1) + C$

27.  $\frac{1}{\log 10} [x (\log x - 1)] + C$

28.  $-3x^{\frac{3}{2}} \cos^3 \sqrt{x} + 6x^{\frac{3}{2}} \sin^3 \sqrt{x} + 6 \cos^3 \sqrt{x} + C$

29.  $-\frac{1}{2} \operatorname{cosec} x \cot x + \frac{1}{2} \log \left| \tan \frac{x}{2} \right| + C$

30.  $x \tan x - \log \sec x - \frac{x}{2} + C$

31.  $xe^x [\log (xe^x) - 1] + C$

32.  $3x \tan^{-1} x - \frac{3}{2} \log |x^2 + 1| + C$

33.  $x \sin x + \cos x + \frac{1}{2} \sin^2 x + C$

**Practice Exercise: 15**

1.  $e^x \tan^{-1} x + C$

2.  $\left( \frac{x-1}{x+1} \right) e^x + C$

3.  $e^x \sin x + C$

4.  $\frac{e^x}{1+x} + C$

5.  $e^x \tan\left(\frac{x}{2}\right) + C$

6.  $\frac{e^x}{x} + C$

7.  $\frac{e^x}{(x-1)^2} + C$

8.  $\frac{1}{x+1} e^x + C$

9.  $e^x \tan x + C$

10.  $x \sin(\log x) + C$

11.  $\frac{x}{\log x + 1} + C$

12.  $\frac{1}{2} e^{2x} \tan x + C$

13.  $e^x \sin^{-1} x + C$

14.  $e^x \left( \log x - \frac{1}{x} \right) + C$

15.  $\frac{x}{\log x} + C$

**Practice Exercise: 16**

13.  $-\frac{1}{3} \log|1 + \tan \theta| + \frac{1}{6} \log|\tan^2 \theta + \tan \theta + 1| + \frac{1}{\sqrt{3}} \tan^{-1} \left( \frac{2 \tan \theta - 1}{\sqrt{3}} \right) + C$

14.  $\frac{1}{2} \log \left| \frac{\log x}{\log(x+2)} \right| + C$

15.  $\log|2 \log x + 1| - \log|3 \log x + 2| + C$

**Practice Exercise: 17**

1.  $\frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{x^2 - 1}{\sqrt{2}x} \right) + C$

2.  $\frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{\tan^2 x - 1}{\sqrt{2} \tan x} \right) + C$

3.  $\sqrt{2} \tan^{-1} \left( \frac{\tan \theta - 1}{\sqrt{2} \tan \theta} \right) + C$

4.  $-\frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{\cot \theta - 1}{\sqrt{2} \cot \theta} \right) - \frac{1}{2\sqrt{2}} \log \left| \frac{\cot \theta + 1 - \sqrt{2} \cot \theta}{\cot \theta + 1 + \sqrt{2} \cot \theta} \right| + C$

5.  $\frac{1}{\sqrt{3}} \tan^{-1} \left( \frac{x^2 - 1}{\sqrt{3}x} \right) - \frac{2}{\sqrt{3}} \tan^{-1} \left( \frac{2x^2 + 1}{\sqrt{3}} \right) + C$

1.  $\frac{11}{4} \log \left| \frac{x+1}{x+3} \right| + \frac{5}{2(x+1)} + C$

2.  $\log \left| \frac{x+1}{x+2} \right| + C$

3.  $x - 5 \log|x-2| + 10 \log|x-3| + C$

4.  $-\frac{1}{3} \tan^{-1} x + \frac{2}{3} \tan^{-1} \frac{x}{2} + C$

5.  $3 \log(2 - \sin \phi) + \frac{4}{2 - \sin \phi} + C$

6.  $\frac{3}{5} \log|x+2| + \frac{1}{5} \log|x^2+1| + \frac{1}{5} \tan^{-1} x + C$

7.  $\frac{1}{4} \log \left| \frac{x-1}{x+1} \right| - \frac{1}{2} \tan^{-1} x + C$

8.  $\frac{1}{n} \log \left| \frac{x^n}{x^n + 1} \right| + C$

9.  $\log \left| \frac{2 - \sin x}{1 - \sin x} \right| + C$

10.  $x + \frac{2}{\sqrt{3}} \tan^{-1} \left( \frac{x}{\sqrt{3}} \right) - 3 \tan^{-1} \left( \frac{x}{2} \right) + C$

11.  $\log|\sec x + \tan x| - 2 \tan \left( \frac{x}{2} \right) + C$

12.  $x + 3 \log|x-4| - 24 \log|x-5| + 30 \log|x-6| + C$

**QUESTION BANK**

1.  $\left(\frac{2}{3}\right)^x \cdot \frac{1}{\log\left(\frac{2}{3}\right)} + \left(\frac{3}{2}\right)^x \cdot \frac{1}{\log\left(\frac{3}{2}\right)} + 2x + C$

2.  $-\cot x - \sin 2x - 2x + C$

3.  $\log|\sin x| - \frac{\sin^6 x}{6} - \frac{3 \sin^2 x}{2} + \frac{3 \sin^4 x}{4} + C$

4.  $\sin^{-1}(\sin x - \cos x) + C$

5.  $-\log|\sin x + \cos x + \sqrt{\sin 2x}| + C$

6.  $\frac{1}{\sin(a-b)} \log \left| \frac{\sin(x-a)}{\sin(x-b)} \right| + C$

7.  $\frac{1}{\sin(a-b)} \log \left| \frac{\cos(x-a)}{\cos(x-b)} \right| + C$

8.  $\frac{1}{\cos(a-b)} \log \left| \frac{\sin(x-a)}{\cos(x-b)} \right| + C$

9.  $\tan^{-1}(\tan^2 x) + C$

$$10. \sqrt{x^2+x} + \frac{1}{2} \log \left| \left( x + \frac{1}{2} \right) + \sqrt{x^2+x} \right| + C$$

$$11. \sqrt{x-x^2} + \frac{1}{2} \sin^{-1}(2x-1) + C$$

$$12. -\frac{2}{a\sqrt{a}} \left[ (a-1) \log |1-\sqrt{ax}| + (2-a)(1-\sqrt{ax}) - \frac{1}{2}(1-\sqrt{ax})^2 \right] + C$$

$$13. \frac{a}{a^2+b^2} x + \frac{b}{a^2+b^2} \log |a \cos x + b \sin x| + C$$

$$18. (x+a) \tan^{-1} \sqrt{\frac{x}{a}} - \sqrt{ax} + C$$

$$14. \tan x + \frac{1}{3} \tan^3 x + C$$

$$19. -2\sqrt{1-x} + \cos^{-1} \sqrt{x} + \sqrt{x}\sqrt{1-x} + C$$

$$15. \tan x + \frac{2}{3} \tan^3 x + \frac{1}{5} \tan^5 x + C$$

$$20. \frac{1}{40} \log \left| \frac{5-4(\sin x - \cos x)}{5+4(\sin x - \cos x)} \right| + C$$

$$16. \frac{1}{n} \log \left| \frac{\sqrt{1+x^n} - 1}{\sqrt{1+x^n} + 1} \right| + C$$

$$21. x \log(x) - \frac{x}{\log x} + C$$

$$17. \frac{1}{2} \left[ x \cos^{-1} x - \sqrt{1-x^2} \right] + C$$

$$22. -\cos \alpha \sin^{-1} \left( \frac{\cos x}{\cos \alpha} \right) - \sin \alpha \cdot \log \left| \sin x + \sqrt{\sin^2 x - \sin^2 \alpha} \right| + C$$

$$23. \sqrt{2} \log \left| \sqrt{2} \cos x + \sqrt{\cos 2x} \right| + \frac{1}{2} \log \left| \frac{\sqrt{\cos 2x} - \cos x}{\sqrt{\cos 2x} + \cos x} \right| + C$$

$$24. \frac{4}{15} \left( 1 - \frac{1}{x^3} \right)^{\frac{5}{4}} + C$$

$$36. -\frac{1}{3} \log |\cos 3x| + \frac{1}{2} \log |\cos 2x| + \log |\cos x| + C$$

$$25. 2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6 \log |1+x^6| + C$$

$$37. \frac{\sec^n x}{n} + C$$

$$26. \frac{-2}{\sin \alpha} \sqrt{\frac{\sin(x+\alpha)}{\sin x}} + C$$

$$38. \frac{1}{\log 5} \left( 5^{x+\tan^{-1} x} \right) + C$$

$$27. -\frac{1}{3} \left( 1 + \frac{1}{x^2} \right)^{\frac{3}{2}} \left[ \log \left( 1 + \frac{1}{x^2} \right) - \frac{2}{3} \right] + C$$

$$39. -\left( 1 + \frac{1}{x^4} \right)^{\frac{1}{4}} + C$$

$$28. -\frac{2}{a} \sqrt{\frac{(a-x)}{x}} + C$$

$$40. \frac{1}{2} (2x^2 - 1) \sin^{-1} \sqrt{x} + \frac{1}{2} \sqrt{x-x^2} + C$$

$$29. \frac{x}{2} |x| + C$$

$$30. x - \frac{x^3}{3} + C$$

$$31. \frac{1}{(\log 2)^3} 2^{2^x} + C$$

$$32. \frac{2}{3} \sin^{-1} \left( \frac{x^{\frac{3}{2}}}{a^{\frac{3}{2}}} \right) + C$$

$$33. 2x \tan^{-1} 3x - \frac{1}{3} \log |1+9x^2| + C$$

$$34. \log |\sec x + \tan x| + \log |\sec x| + C$$

$$35. (x+b) \cos(a-b) + \sin(a-b) \log |\sin(x+b)| + C$$

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