Matrix and Determinants

1/2Marks

1. Under what conditions is the matrix equation $A^2 - B^2 = (A-B)(A+B)$ is true.
2. Write the order of matrix B if A is any matrix of order m×n such that AB and BA both are defined.
3. Give an example of two non-zero matrices A and B such that $AB = 0$ but $BA \neq 0$.
4. If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ and $A^2 = I$, find value of $\alpha^2 + \beta\gamma$.
5. If the following matrix is skew symmetric , find the value of *a, b, c* : $A = \begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$
6. Give an example of a matrix which is both symmetric and skew symmetric.
7. $\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} = P + Q$, where *P* is symmetric and *Q* is skew symmetric matrix, find the matrix *P* and *Q*.
8. If A is a square matrix then write the value of $A(adjA)$.
9. If A is a square matrix of order 3 and $|A| = 5$, find the value $|-3A|$.
10. For what value of *k*, the matrix $\begin{bmatrix} 2 & k \\ 3 & 5 \end{bmatrix}$ has no inverse.

11. A, B, C are three non-zero matrices of same order, then find the condition on A such that $AB = AC \Rightarrow B = C$.
12. Let A be a non-singular matrix of order 3×3, such that $|adjA| = 100$, find $|A|$.
13. Using determinants find the value of k for which the following system of equation has unique solution $2x - 5y = 26$, $3x + ky = 5$.
14. Find the value of $a_{23} + a_{32}$ in the matrix $A = [a_{ij}]_{3\times3}$ where $A = \begin{cases} |2i - j| & if\ i > j \\ -i + 2j + 3 & if\ i \leq j \end{cases}$
15. If B be 4×5 type matrix then, what is the number of elements in third column.
16. Find the value of P, such that the matrix $\begin{bmatrix} -1 & 2 \\ 4 & P \end{bmatrix}$ is singular.
17. Find the value of *x* such that points $(0,2)$, $(1,x)$ and $(3,1)$ are collinear.
18. Area of a triangle with vertices $(k,0)$, $(1,1)$, $(0,3)$ is 5 units. Find values of k.
19. If $A = 2B$ where A and B are square matrices of order 3×3 and $|B| = 5$ what is $|A|$.
20. What is the condition that a system of equation $AX = B$ has no solution.
21. If A is a non-singular matrix of order 3 and $|A| = -3$, find $|adjA|$.
22. Given a square matrix A of order 3×3 such that $|A| = 12$, find the value of $|A(adjA)|$.

23. If $A=\begin{bmatrix}2 & -1\\3 & 4\end{bmatrix}$, find $\left|\left(A^{-1}\right)^{-1}\right|$.

24. If a matrix A has 11 elements, what is its possible order?

25. What is the number of all possible matrices of order 2×3 with each entry 0, 1 or 2.

26. What is the value of $|3I|$, where I is identity matrix of order 3?

27. If A is a square matrix of order 3 such that $|adjA|=64$, find $\left|A^T\right|$.

28. If $A=\begin{bmatrix}a & 2\\2 & a\end{bmatrix}$ and $|A|^3=125$, then find *a* .

29. Find possible values of *x* if $\begin{vmatrix}16 & 3\\5 & 2\end{vmatrix}=\begin{vmatrix}2x & 3\\5 & x\end{vmatrix}$

30. Construct a 3×3 matrix whose elements a_{ij} are given by in each of the following:

(i) $a_{ij}=\dfrac{i+3j}{2}$ (ii) $a_{ij}=\begin{cases}i-j, if\ i\geq j\\ i+j, if\ i<j\end{cases}$ (iii) $a_{ij}=\dfrac{1}{2}|-3i+j|$

31. Find x and y if $2x+3y=\begin{bmatrix}2 & 3\\4 & 0\end{bmatrix}$ and $3x+2y=\begin{bmatrix}2 & -2\\-1 & 5\end{bmatrix}$

32. If each element of 1st row and 3rd row of a determinant of third order with value α is multiplied by 2 and 5 respectively, then find the value of the determinant.

33. Given $A=\begin{bmatrix}a_{11} & a_{12}\\a_{21} & a_{22}\end{bmatrix}$ such that $|A|=-10$, Find $a_{11}.c_{11}+a_{12}.c_{12}$

4/6 Marks

1. If $A=\begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$, find k if $A^2=kA-2I$.

2. If $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\-\sin\alpha & \cos\alpha\end{bmatrix}$, then show that $A^2=\begin{bmatrix}\cos 2\alpha & \sin 2\alpha\\-\sin 2\alpha & \cos 2\alpha\end{bmatrix}$.

3. Evaluate: $\begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}4\\4\\4\end{bmatrix}$.

4. Let $P=\begin{bmatrix}\cos\alpha & \sin\alpha\\-\sin\alpha & \cos\alpha\end{bmatrix}$, then show that $PP'=I$, where I is a unit matrix of order 2.

5. Express the matrix $\begin{bmatrix}6 & 1 & -5\\-2 & -5 & 4\\-3 & 3 & -1\end{bmatrix}$ as the sum of symmetric and skew-symmetric matrix.

6. Show that matrix $A=\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}$ satisfies the equation $A^2-4A+I=O$ and hence find A^{-1}.

7. Find the equation of the line joining the points $(1,2)$ and $(3,6)$ using determinants.

8. Let $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$, using principle of mathematical induction prove that:

$A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}$.

9. If $f(x)=x^2-5x+7$, find $f(A)$ if $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$.

10. Find x if $\begin{bmatrix}x & -5 & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}\begin{bmatrix}x\\ 4\\ 1\end{bmatrix}=0$.

11. Find matrix P satisfying the matrix equation. $\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}P\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$.

12. If $P(x_1,y_1),Q(x_2,y_2),R(x_3,y_3)$ are the vertices of an equilateral triangle whose each side is equal to 'a', then prove that: $\begin{vmatrix}x_1 & y_1 & 2\\ x_2 & y_2 & 2\\ x_3 & y_3 & 2\end{vmatrix}^2=3a^4$.

13. If $A=\begin{bmatrix}3 & 2\\ 1 & 1\end{bmatrix}$, find the values of a and b such that $A^2+Aa+bI=O$. Hence find A^{-1}.

Using properties of determinants, prove each of the following: (14 – 38)

14. $\begin{vmatrix}a & 1 & b+c\\ b & 1 & c+a\\ c & 1 & a+b\end{vmatrix}=0$

15. $\begin{vmatrix}1 & a & a^3\\ 1 & b & b^3\\ 1 & c & c^3\end{vmatrix}=(a-b)(b-c)(c-a)(a+b+c)$.

16. $\begin{vmatrix}1+a & 1 & 1\\ 1 & 1+b & 1\\ 1 & 1 & 1+c\end{vmatrix}=abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$.

17. $\begin{vmatrix}x-y-z & 2x & 2x\\ 2y & y-z-x & 2y\\ 2z & 2z & z-x-y\end{vmatrix}=(x+y+z)^3$.

18. $\begin{vmatrix}a & b & c\\ a-b & b-c & c-a\\ b+c & c+a & a+b\end{vmatrix}=a^3+b^3+c^3-3abc$.

19. $\begin{vmatrix} (y+z)^2 & x^2 & x^2 \\ y^2 & (z+x)^2 & y^2 \\ z^2 & z^2 & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3.$

20. $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(x+a+b+c).$

21. $\begin{vmatrix} 0 & ab^2 & ac^2 \\ a^2b & 0 & bc^2 \\ a^2c & b^2c & 0 \end{vmatrix} = 2a^3b^3c^3.$

22. $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3.$

23. $\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a).$

24. $\begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix} = (1-x^2)\begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}.$

25. $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc.$

26. $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3.$

27. $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}.$

28. $\begin{vmatrix} x^2+1 & xy & zx \\ xy & y^2+1 & yz \\ zx & yz & z^2+1 \end{vmatrix} = 1+x^2+y^2+z^2.$

29. $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3.$

30. $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$.

31. $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1$.

32. $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$.

33. $\begin{vmatrix} \sin^2 A & \sin A.\cos A & \cos^2 A \\ \sin^2 B & \sin B.\cos B & \cos^2 B \\ \sin^2 C & \sin C.\cos C & \cos^2 C \end{vmatrix} = -\sin(A-B)\sin(B-C)\sin(C-A)$.

34. $\begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix} = 0$.

35. $\begin{vmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix} = 16(3x+4)$.

36. $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$.

37. $\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$.

38. $\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix} = (x+y+z)(x-z)^2$.

39. Using elementary transformation find the inverse of each of the following matrices

(i) $\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & -1 & 3 \\ 1 & 3 & -1 \\ 3 & 2 & 1 \end{bmatrix}$

40. If a, b, c are in A.P. prove that: $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$.

41. If a, b, c are in A.P. evaluate: $\begin{vmatrix} 2y+4 & 5y+7 & 8y+a \\ 3y+5 & 6y+8 & 9y+b \\ 4y+6 & 7y+9 & 10y+c \end{vmatrix}$.

42. If a, b, c are respectively p^{th}, q^{th} and r^{th} term of G.P. prove that $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$.

43. If x, y, z are different and $A = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then show that $(1+xyz)=0$.

44. Given $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ find $(AB)^{-1}$.

45. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equation

$x - y + 2z = 1,\ 2y - 3z = 1,\ 3x - 2y + 4z = 2$

46. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$, find A^{-1} and use it to solve the system of equations

$x + y + 2z = 0,\ x + 2y - z = 9,\ x - 3y + 3z + 14 = 0$.

47. For what value of θ, the system of equations:

$(\sin 3\theta)x - y + z = 0,\ (\cos 2\theta)x + 4y + 3z = 0,\ 2x + 7y + 7z = 0$ has non-trivial solution.

48. Without expanding prove the determinant prove that: $\begin{vmatrix} \frac{1}{a} & a^2 & bc \\ \frac{1}{b} & b^2 & ca \\ \frac{1}{c} & c^2 & ab \end{vmatrix} = 0$.

49. Using matrix method solve the system of linear equation:

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4,\ \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1,\ \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$

50. If $\begin{vmatrix} x & b & c \\ a & y & c \\ a & b & z \end{vmatrix} = 0$, then find the value of $\frac{x}{x-a} + \frac{y}{y-b} + \frac{z}{z-c}$

51. Prove that the product of matrices $\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and $\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ is null matrix, when θ and ϕ differ by an odd multiple of $\frac{\pi}{2}$.

52. If $a+b+c=0$ and $\begin{vmatrix} a-x & a & b \\ c & b-x & a \\ b & c & c-x \end{vmatrix}=0$, then prove that either $x=0$ or $x=\pm\sqrt{\frac{3}{2}\left(a^2+b^2+c^2\right)}$.

53. If $A=\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$ prove that $A^n=\begin{bmatrix} a^n & \dfrac{b\left(a^n-1\right)}{a-1} \\ 0 & 1 \end{bmatrix}, n\in N$

54. For any square matrix verify $A(adjA)=|A|I$.

55. If $f(x)=\begin{vmatrix} \cos^2 x & \cos x.\sin x & -\sin x \\ \cos x.\sin x & \sin^2 x & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix}$ then find $f(x)$ $[Ans.:1]$

56. Prove that: $\begin{vmatrix} \sqrt{13}+\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{26} & 5 & \sqrt{10} \\ 3+\sqrt{65} & \sqrt{15} & 5 \end{vmatrix}=-5\sqrt{3}\left(5-\sqrt{6}\right)$

57. If $a^2+b^2+c^2=1$, then prove that

$$:\begin{vmatrix} a^2+\left(b^2+c^2\right)\cos\theta & ab(1-\cos\theta) & ac(1-\cos\theta) \\ ba(1-\cos\theta) & b^2+\left(c^2+a^2\right)\cos\theta & bc(1-\cos\theta) \\ ca(1-\cos\theta) & cb(1-\cos\theta) & c^2+\left(a^2+b^2\right)\cos\theta \end{vmatrix}=\cos^2\theta$$

58. Prove that: $\begin{vmatrix} 1 & bc & bc(b+c) \\ 1 & ca & ca(c+a) \\ 1 & ab & ab(a+b) \end{vmatrix}=0$

59. Prove that: $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix}=\left(a^2+b^2+c^2\right)(a+b+c)$

60. Prove that: $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix}=0$

61. Prove that: $\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{vmatrix} = 4abc$

62. Prove that: If $a \neq b \neq c$ and $\begin{vmatrix} a & a^3 & a^4-1 \\ b & b^3 & b^4-1 \\ c & c^3 & c^4-1 \end{vmatrix} = 0$, then prove that: $abc(ab+bc+ca) = a+b+c$

63. If $\begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix} = 0$, and $x \neq 0$ then prove that: $x = -\left(a^2+b^2+c^2\right)$

64. Prove that: $\begin{vmatrix} {}^8C_3 & {}^9C_5 & {}^{10}C_7 \\ {}^8C_4 & {}^9C_6 & {}^{10}C_8 \\ {}^9C_4 & {}^{10}C_6 & {}^{11}C_8 \end{vmatrix} = 0$

65. If $f(x) = \begin{vmatrix} 1 & 2(x-1) & 3(x-1)(x-2) \\ (x-1) & (x-1)(x-2) & (x-1)(x-2)(x-3) \\ x & (x-1)x & (x-1)(x-2)x \end{vmatrix}$, then find $f(41)$ $[Ans.: 0]$

66. If 4^n is a factor of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ {}^nC_1 & {}^{n+4}C_1 & {}^{n+8}C_1 \\ {}^nC_2 & {}^{n+4}C_2 & {}^{n+8}C_2 \end{vmatrix}$, the maximum value of n is $[Ans.: n = 3]$

67. Prove that: $\begin{vmatrix} \sqrt{x}+\sqrt{y} & 2\sqrt{z} & \sqrt{z} \\ \sqrt{yz}+\sqrt{2x} & z & \sqrt{2z} \\ y+\sqrt{xz} & \sqrt{yz} & z \end{vmatrix} = z\left(\sqrt{2}y - z\sqrt{y}\right)$

68. If $\begin{vmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{vmatrix}$, *then* $f(3x) - f(x) =$ $\left[Ans.: 6x\lambda^2\right]$

69. If [ ] denotes the greatest integer less than or equal to the real number under consideration and $-1 \leq x < 0;\ 0 \leq y < 1;\ 1 \leq z < 2$, then the value of determinant $\begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$

$[Ans.: [z]]$

70. Write the element a_{23} of a 3×3 matrix $A = (a_{ij})$ whose elements a_{ij} are given by $a_{ij} = \frac{|i-j|}{2}$

(cbse2015)

71. If $f(x)=\begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$ using properties of determinants find the value of $f(2x)-f(x)$

(cbse2015)

72. Three schools A, B and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold hand made fans, mats and plates from recycled material at cost of ₹25, ₹100 and ₹50 each. The number of articles sold are given below:

School / Article	**A**	**B**	**C**
Hand-fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Find the funds collected by each school separately by selling the above articles. Also find the total funds collected for the purpose. Write one value generated by above situation.

(cbse2015)

ANSWERS

1Mark

1. $AB=BA$
2. n×m
3. $A=\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ $B=\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
4. 1
5. $a=-2, b=0, c=-3$
6. Null Matrix
7. $P=\begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}$ $Q=\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$
8. $|A|I$
9. -135
10. $k=\frac{10}{3}$
11. $|A|\neq 0$
12. $|A|=\pm 10$
13. $k\neq -\frac{15}{2}$
14. 11

15. 4

16. -8

17. $x = \frac{5}{3}$

18. $-\frac{7}{2}$ or $\frac{13}{2}$

19. 40

20. $|A| = 0, (adjA).B \neq 0$

21. 9

22. 1728

23. 11

24. 1×11, 11×1

25. 729

26. 27

27. ± 8

28. ± 3

29. ± 4

30. (i) $\begin{bmatrix} 2 & \frac{7}{2} & 5 \\ \frac{5}{2} & 4 & \frac{11}{2} \\ 3 & \frac{9}{2} & 6 \end{bmatrix}$ (ii) $\begin{bmatrix} 0 & 3 & 4 \\ 1 & 0 & 5 \\ 2 & 1 & 0 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & \frac{1}{2} & 0 \\ \frac{5}{2} & 2 & \frac{3}{2} \\ 4 & \frac{7}{2} & 3 \end{bmatrix}$

31. $X = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix} Y = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$

32. 10α

33. -10

4/6Marks

1. $k = 1$

3. $[12]$

6. $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

7. $2x - y = 0$

9. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

10. $\pm 4\sqrt{3}$

11. $\begin{bmatrix} 25 & 15 \\ -37 & -22 \end{bmatrix}$

13. $a = -4, b = 1, A^{-1} = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$

39. (i) Does not exists(ii) $\begin{bmatrix} -\frac{5}{7} & -1 & \frac{8}{7} \\ \frac{4}{7} & 1 & -\frac{5}{7} \\ 1 & 1 & -1 \end{bmatrix}$

45. $x = 0, y = 5, z = 3$

46. $A^{-1} = -\frac{1}{11}\begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix} x = 1, y = 3, z = -2$

47. $\theta = n\pi \; or \; \theta = n\pi + (-1)^n \frac{\pi}{6}, n \in I$

49. $x = 2, y = 3, z = 5$

ingeniousinfinity
Believe in knowledge . . .