Three Dimensional Geometry

1/2 Marks

1. The cartesian equation of a line AB is $\frac{2x-1}{\sqrt{3}} = \frac{y+2}{2} = \frac{z-3}{3}$. Find the direction cosines of a line parallel to AB.
2. Write the vector equation of the following line : $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$.
3. Write the position vector of the mid-point of the vector joining the points $P(2,3,4)$ and $Q(4,1,-2)$.
4. Write the intercept cut off by the plane $2x + y - z = 5$ on $x - axis$.
5. What are the direction cosines of a line, which makes equal angles with coordinate axis?
6. If a line has direction ratios 2, -1, -2, then what are its direction cosines?
7. Find the distance of the plane $3x - 4y + 12z = 3$ from the origin.
8. Write the direction cosines of a line parallel to z-axis.
9. Find the vector normal to the plane $\vec{r}.\left(3\hat{\imath} - 7\hat{k}\right) + 5 = 0$.
10. A line in $xy - plane$ makes angle $\frac{\pi}{6}$ with $x - axis$. Find the direction ratios and direction cosines of the line.
11. Find the coordinates of a point, where the line $\frac{x+2}{1} = \frac{y-5}{3} = \frac{z+1}{5}$ cuts $yz - plane$.

4/6 Marks

12. Find the coordinates of the foot of the perpendicular drawn from the point $A(1,8,4)$ to the line joining the points $B(0,-1,3)\ and\ C(2,-3,-1)$.
13. Find the Cartesian as well as the vector equation of the planes passing through the intersection of the planes $\vec{r}.(2\hat{\imath} + 6\hat{\jmath}) + 12 = 0$ and $\vec{r}.\left(3\hat{\imath} - \hat{\jmath} + 4\hat{k}\right) = 0$, which are at unit distance from the origin.
14. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ inetsect. Also, find the point of intersection.
15. Find the perpendicular distance of the point $(1,0,0)$ from the line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$.
16. Find the equation of the line passing through the point $P(-1,3,-2)$ and perpendicular to the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$.
17. Find the equation of the plane passing through the points $P(1,-1,2)\ and\ Q(2,-2,2)$ and perpendicular to the plane $6x - 2y + 2z = 9$.
18. Find the vector equation and Cartesian equation of the line passing through the points $P(0,1,-2)\ and\ Q(-1,-1,-3)$. Also prove that it passes through the point R whose position vector is $-3\hat{\imath} - 5\hat{\jmath} - 5\hat{k}$.
19. Find the equation of the plane passing through the line of intersection of the planes $x - 2y + z = 1$ and $2x + y + z = 8$ and parallel to the line with direction ratios $1, 2, 1$. Also find the perpendicular distance of the point $P(3,1,2)$ from this plane.
20. Find the equation of the line drawn perpendicular from the point $P(1,6,3)$ to the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also find the perpendicular distance of the given line from the point P.
21. Find the vector and Cartesian equation of a plane passing through the point $(1,2,3)$ and perpendicular to the line with direction ratios $2, 3, -4$.

22. Find the equation of the plane passing through the points $(1, 2, 3)$ *and* $(0, -1, 0)$ and parallel to the line
$$\frac{x-1}{2} = \frac{y-1}{2} = \frac{z}{-3}.$$
23. Find the coordinates of the point where the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4}$ meet the plane $x + y + 4z = 6$.
24. Find the image of the point $(1, 2, 3)$ in the plane $x + 2y + 4z = 38$.
25. Find the equation of the plane which is perpendicular to the plane $5x + 3y + 6z + 8 = 0$ and which contains the line of intersection of the planes $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$.
26. Find the shortest distance between the following lines: $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$.
27. Find the point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance $3\sqrt{2}$ from the point $(1, 2, 3)$.
28. Find the length and foot of perpendicular drawn from the point $(2, -1, 5)$ to the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$
29. From the point $P(1, 2, 4)$ a perpendicular is drawn on the plane $2x + y - 2z + 3 = 0$. Find the equation, the length and coordinates of the foot of the perpendicular.
30. Find the distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$.
31. Find the equation of the line passing through the point $P(4, 6, 2)$ and point of intersection of the line
$\frac{x-1}{3} = \frac{y}{2} = \frac{z+1}{7}$ and the plane $x + y - z = 8$
32. Find the distance of the point $(-2,\ 3\ , -4)$ from the line from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$.
33. Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}; \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar. Also find the equation of the plane containing the lines.
34. Find the Cartesian equation of the plane passing through the points $A(0, 0, 0)$ and $B(3, -1, 2)$ and parallel to the line $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$.
35. Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point $P(1, 3, 3)$.
36. Find the coordinate of the foot of the perpendicular and perpendicular distance of the point $P(3, 2, 1)$ from the plane $2x - y + z + 1 = 0$. Find also the image of the point in the plane.
37. Find the coordinates of the point where the line through the points $A(3, 4, 1)$ and $B(5, 1, 6)$ crosses $XY - plane$.
38. If the line $\frac{x-1}{-3} = \frac{y-2}{-2k} = \frac{z-3}{2}$ and $\frac{x-1}{k} = \frac{y-2}{1} = \frac{z-3}{5}$ are perpendicular, find the value of k and hence find the equation of plane containing these lines.
39. Find the vector equation of the plane passing through the points $(2, 1, -1)$ and $(-1, 3, 4)$ and perpendicular to the plane $x - 2y + 4z = 10$. Also show that the plane thus obtained contains the line $\vec{r} = -\hat{\imath} + 3\hat{\jmath} + 4\hat{k} + \lambda(3\hat{\imath} - 2\hat{\jmath} - 5\hat{k})$.
40. If a line makes angles α, β, γ with $x, y\ and\ z - axis$ respectively, prove that
$(i)\ \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$
$(ii)\ \cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$
41. A line make angles $60°\ and\ 45°$ with $x\ and\ y - axis$ respectively, find the angle which it makes with $z - axis$.
42. Show that the line joining the points $(4, 7, 8), (2, 3, 4)$ is parallel to the line through the points $(-1, -2, 1), (1, 2, 5)$.
43. Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect each other. Find the point of intersection also.
44. Find the projection of the line segment joining the points $(10, 3, 6)\ and\ (0, 1, 5)$on the line joining the points $(2, 1, 5)\ and\ (4, 0\ , 3)$.

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45. Find the equations of the planes through the intersection of the planes $x + 3y + 6 = 0$ and $3x - y - 4z = 0$ whose perpendicular distance from the origin is equal to 1.
46. Prove that the angle between any two diagonals of a cube is $\cos^{-1}\frac{1}{3}$
47. Find distance between the parallel planes $2x - y + 3z - 4 = 0$ and $6x - 3y + 9z + 13 = 0$.
48. Show that the equation of the plane, which meets the axes in A, B *and* C and given centroid of triangle ABC is the point (α, β, γ) is $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$.
49. Find the perpendicular distance of point $(2, 3, 4)$ from the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.
50. The cartesian equations of a line are $3x + 1 = 6y - 2 = 1 - z$. Find the fixed point through which it passes, its direction ratios and also its vector equation.

Answers

1. $\sqrt{3}, 4, 6$
2. $\vec{r} = (5\hat{\imath} - 4\hat{\jmath} + 6\hat{k}) + \lambda(3\hat{\imath} + 7\hat{\jmath} - 2\hat{k})$
3. $3\hat{\imath} + 2\hat{\jmath} + \hat{k}$
4. $\frac{5}{2}$
5. $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$
6. $\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}$
7. $\frac{3}{13} units$
8. $0, 0, 1$
9. $3\hat{\imath} - 7\hat{k}$
10. $\sqrt{3}, 1, 0$; $\frac{\sqrt{3}}{2}, \frac{1}{2}, 0$
11. $(0, 11, 9)$
12. $\left(-\frac{5}{3}, \frac{2}{3}, \frac{19}{3}\right)$
13. $\vec{r} = (2\hat{\imath} + \hat{\jmath} + 2\hat{k}) + 3 = 0;$
    $2x + y + 2z + 3 = 0$
    $\vec{r} = (\hat{\imath} - 2\hat{\jmath} + 2\hat{k}) - 3 = 0;$
    $x - 2y + 2z - 3 = 0$
14. $(-1, -1, -1)$
15. $2\sqrt{6} units$
16. $\frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}$
17. $x + y - 2z + 4 = 0$
18. $\vec{r} = (\hat{\jmath} - 2\hat{k}) + \lambda(\hat{\imath} + 2\hat{\jmath} + \hat{k})$
19. $9x - 8y + 7z - 21 = 0$
20. $\frac{x-1}{0} = \frac{y-6}{-3} = \frac{z-3}{2}$; $\sqrt{13}$
21. $\vec{r} = (2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}) + 4 = 0;$
    $2x + 3y - 4z + 4 = 0$
22. $6x - 3y + z - 3 = 0$
23. $(1, 1, 1)$
24. $(3, 6, 11)$
25. $51x + 15y - 50z + 173 = 0$
26. $2\sqrt{29}$
27. $(-2, -1, 3); \left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$
28. $\sqrt{14}; (1, 2, 3)$
29. $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-1}{-2}$; $\frac{1}{3}$; $\left(\frac{11}{9}, \frac{19}{9}, \frac{34}{9}\right)$
30. 1
31. $\frac{x-4}{1} = \frac{y-6}{1} = \frac{z-2}{2}$
32. $\frac{17}{2}$
33. $x - 2y + z = 0$
34. $x - 19y - 11z = 0$
35. $(-2, -1, 3), (4, 3, 7)$
36. $(1, 3, 0), (-1, 4, -1)$
37. $\left(\frac{13}{5}, \frac{23}{5}, 0\right)$
38. $k = 2, 22x - 19y - 5z + 31 = 0$
39. $\vec{r}.(18\hat{\imath} + 17\hat{\jmath} + 4\hat{k}) - 49 = 0$
41. $60°, 120°$
43. $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$
44. $\frac{16}{3}$
45. $x - 2y - 2z - 3 = 0; 2x + y - 2z + 3 = 0$
47. $\frac{25\sqrt{14}}{42}$
49. $\frac{1}{49}\sqrt{44541}$
50. $\left(-\frac{1}{3}, \frac{1}{3}, 1\right); 2, 1, -6;$
    $\vec{r} = \left(-\frac{1}{3}\hat{\imath} + \frac{1}{3}\hat{\jmath} + \hat{k}\right) + \lambda(2\hat{\imath} + \hat{\jmath} - 6\hat{k})$

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