Vector

1. Find a unit vector in direction of $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + 6\hat{k}$.
2. Find the angle between the vectors $\vec{a} = \hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + \hat{\jmath} - \hat{k}$
3. For what value of λ are the vectors $\vec{a} = 2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ perpendicular to each other?
4. If $\vec{a} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\vec{b} = 3\hat{\imath} + \hat{\jmath} - 5\hat{k}$, find a unit vector in the direction of $\vec{a} - \vec{b}$.
5. If $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $\vec{a}.\vec{b} = 3$, find the angle between $\vec{a}$ and $\vec{b}$.
6. If $\vec{a} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{b} = 2\hat{\imath} + 4\hat{\jmath} + 9\hat{k}$, find a unit vector parallel to $\vec{a} + \vec{b}$.
7. If $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and angle between $\vec{a}$ and $\vec{b}$ is $60°$, find $\vec{a}.\vec{b}$.
8. Find a vector in direction of $\vec{a} = \hat{\imath} - 2\hat{\jmath}$ whose magnitude is 7.
9. Find the projection of $\vec{a}$ on $\vec{b}$ if $\vec{a}.\vec{b} = 8$ and $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$.
10. Write the value of p for which $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$ and $\vec{b} = \hat{\imath} + p\hat{\jmath} + 3\hat{k}$ are parallel vectors.
11. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively and when $|\vec{a} \times \vec{b}| = \sqrt{3}$.
12. Find the value of p, if $(2\hat{\imath} + 6\hat{\jmath} + 27\hat{k}) \times (\hat{\imath} + 3\hat{\jmath} + p\hat{k}) = \vec{0}$
13. Write the direction cosines of a line equally inclined to three coordinate axes.
14. If p is a unit vector and $(\vec{x} - \vec{p}).(\vec{x} + \vec{p}) = 80$ then find $|\vec{x}|$.
15. What is the cosine of the angle which the vector $\sqrt{2}\hat{\imath} + \hat{\jmath} + \hat{k}$ make with y-axis?
16. Find the value of λ, if $(2\hat{\imath} + 6\hat{\jmath} + 14\hat{k}) \times (\hat{\imath} - \lambda\hat{\jmath} + 7\hat{k}) = \vec{0}$
17. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$then what is the angle between $\vec{a}$ and $\vec{b}$?
18. Vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = \frac{2}{3}$ and $(\vec{a} \times \vec{b})$ is a unit vector. Write the angle between $\vec{a}$ and $\vec{b}$.
19. For what value of p, is $(\hat{\imath} + \hat{\jmath} + \hat{k})p$ a unit vector?
20. For what value of a the vectors $2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $a\hat{\imath} + 6\hat{\jmath} - 8\hat{k}$ are collinear?
21. Write the direction cosines of the vector $-2\hat{\imath} + \hat{\jmath} - 5\hat{k}$.
22. Write the position vector of the mid-point of the vector joining the points $P(2,3,4)$ *and* $Q(4,1,-2)$.
23. Find $'\lambda'$ when the projections of $\vec{a} = \lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}$ on $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ is 4 units.
24. Write the value of $(\hat{\imath} \times \hat{\jmath}).\hat{k} + \hat{\imath}.\hat{\jmath}$.
25. Write the value of $(\hat{\imath} \times \hat{\jmath}).\hat{k}(\hat{\jmath} \times \hat{k}).\hat{\imath}$
26. Write the value of the area of the parallelogram determine by the vectors $2\hat{\imath}$ and $3\hat{\jmath}$.
27. Show that $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$, if $\vec{a}$ and $\vec{b}$ are along adjacent sides of a rectangle.
28. Given $\overrightarrow{AB} = 3\hat{\imath} - \hat{\jmath} - 5\hat{k}$ are coordinates of the terminal point are $(0,1,3)$. Find coordinates of the initial point.
29. Prove that $|\vec{a}.\vec{b}| \leq |\vec{a}||\vec{b}|$.
30. Show that the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ is equally inclined to axes.
31. If $\vec{a}$ and $\vec{b}$ are nonzero vectors, such that $\vec{a} \times \vec{b} = \vec{0}$. How are $\vec{a}$ and $\vec{b}$ related.

32. If $\vec{a}$, $\vec{b}$, $\vec{c}$ be position vectors of vertices A, B, C of parallelogram $ABCD$, find the position vector of D.
33. If $\vec{a}$, $\vec{b}$ are the position vectors of the points $(1,-1),(-2,m)$. Find the value of $'m'$ for which $\vec{a}$ and $\vec{b}$ are collinear.
34. If vectors $2\hat{\imath}-5\hat{\jmath}+b\hat{k}$ and $2\hat{\imath}+a\hat{\jmath}+4\hat{k}$ are parallel, find the value of a and b.
35. If the position vector $\vec{a}$ of point $(-5,\lambda)$ be such that $|\vec{a}|=13$ find λ.
36. What are the vertical and horizontal components of a vector $\vec{a}$ of magnitude 5 making an angle of $150°$ with direction of X-axis.
37. What is $a \in R$ such that $|a\vec{x}|=1$ where $\vec{x}=\hat{\imath}-2\hat{\jmath}+2\hat{k}$?
38. When $|\vec{x}+\vec{y}|=|\vec{x}|+|\vec{y}|$?
39. If A is the point $(4,5)$ and vector $\overrightarrow{AB}$ has components 2 and 6 along X-axis and Y-axis respectively then write point B.
40. What is the point of trisection of PQ nearer P if position of P and Q are $3\hat{\imath}+3\hat{\jmath}-4\hat{k}$ and $9\hat{\imath}+8\hat{\jmath}-10\hat{k}$ respectively.
41. What are the direction cosines of a vector equiangular with coordinate axes?
42. What is the angle which the vector $3\hat{\imath}-6\hat{\jmath}+2\hat{k}$ makes with X-axis?
43. If $|\vec{a}|=2$, $|\vec{b}|=2\sqrt{3}$ and $\vec{a}\perp\vec{b}$ what is the value of $|\vec{a}+\vec{b}|$?
44. What is the angle between $\vec{a}$ and $\vec{b}$, if $|\vec{a}-\vec{b}|=|\vec{a}+\vec{b}|$?
45. What is the area of a parallelogram whose diagonals are given by vectors $2\hat{\imath}+\hat{\jmath}-2\hat{k}$ and $-\hat{\imath}+2\hat{k}$?
46. If $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$ are the usual three mutually perpendicular unit vectors then what is the value of $\hat{\imath}.(\hat{\jmath}\times\hat{k})+\hat{\jmath}.(\hat{\imath}\times\hat{k})+\hat{k}.(\hat{\jmath}\times\hat{\imath})$?
47. What is the angle between $\vec{x}$ and $\vec{y}$ if $\vec{x}.\vec{y}=|\vec{x}\times\vec{y}|$?
48. Write a unit vector in $XY-plane$, making angle of $30°$ with the positive direction of $X-axis$.
49. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $(\vec{a}+2\vec{b})$ is perpendicular to $(5\vec{a}-4\vec{b})$, then what is the angle between $\vec{a}$ and $\vec{b}$

4 marks

50. Dot-product of a vector with vectors $3\hat{\imath}-5\hat{k}$, $2\hat{\imath}+7\hat{\jmath}$, $\hat{\imath}+\hat{\jmath}+\hat{k}$ are respectively $-1, 6$ and 5. Find the vector
51. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, show that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$. Also interpret the result geometrically.
52. Show that $[\vec{a}+\vec{b}\quad\vec{b}+\vec{c}\quad\vec{c}+\vec{a}]=2[\vec{a}\quad\vec{b}\quad\vec{c}]$
53. If $\vec{a}=3\hat{\imath}-\hat{\jmath}+2\hat{k}$, $\vec{b}=2\hat{\imath}+\hat{\jmath}-\hat{k}$ and $\vec{c}=\hat{\imath}-2\hat{\jmath}+2\hat{k}$, then show that $(\vec{a}\times\vec{b})\times\vec{c}\neq\vec{a}\times(\vec{b}\times\vec{c})$
54. For the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ prove that $\vec{a}\times(\vec{b}+\vec{c})+\vec{b}\times(\vec{c}+\vec{a})+\vec{c}\times(\vec{a}+\vec{b})=\vec{0}$
55. Find a unit vector perpendicular to the plane ABC, where A, B, C are the points $(3,-1,2),(1,-1,-3),(4,-3,1)$ respectively.
56. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, prove that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$, provided $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$.
57. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors of equal magnitude, prove that the angle which $(\vec{a}+\vec{b}+\vec{c})$ makes with any of the vectors $\vec{a}$, $\vec{b}$ or $\vec{c}$ is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$.
58. Find the projection of $\vec{b}+\vec{c}$ on $\vec{a}$, where $\vec{a}=2\hat{\imath}-2\hat{\jmath}+\hat{k}$, $\vec{b}=\hat{\imath}+2\hat{\jmath}-2\hat{k}$ and $\vec{c}=2\hat{\imath}-\hat{\jmath}+4\hat{k}$.
59. If $\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k}$ and $\vec{b}=\hat{\jmath}-\hat{k}$, find a vector $\vec{c}$ such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}.\vec{c}=3$.

60. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=3$, $|\vec{b}|=5$ and $|\vec{c}|=7$, show that angle between $\vec{a}$ and $\vec{b}$ is $60°$.
61. Show that area of parallelogram having diagonals $3\hat{\imath}+\hat{\jmath}-2\hat{k}$ and $\hat{\imath}-3\hat{\jmath}+4\hat{k}$ is $5\sqrt{3}\ square\ units.$
62. Define the scalar and vector product of two vectors $\vec{a}$ and $\vec{b}$. If for three non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$; $\vec{a}.\vec{b}=\vec{a}.\vec{c}$ and $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$, then show that $\vec{b}=\vec{c}$.
63. Find a unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=3\hat{\imath}+2\hat{\jmath}+2\hat{k}$ and $\vec{b}=\hat{\imath}+2\hat{\jmath}-2\hat{k}$
64. If $\vec{a}$ and $\vec{b}$ are any two vectors, then give the geometrical interpretation of the relation $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$.
65. Show that angle between any two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.
66. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are position vectors of A, B and C of a triangle ABC, show that area of a triangle is $\frac{1}{2}|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|$
67. If $|\vec{a}|=2$, $|\vec{b}|=3$, $\vec{a}.\vec{b}=4$ find $|\vec{a}-\vec{b}|$
68. If the sum of two unit vectors is a unit vector, prove that magnitude of their difference is $\sqrt{3}$.
69. For any vector $\vec{a}$, show that $\vec{a}=(\vec{a}.\hat{\imath}).\hat{\imath}+(\vec{a}.\hat{\jmath}).\hat{\jmath}+(\vec{a}.\hat{k}).\hat{k}$.
70. Prove that for non-zero vectors $\vec{a}$ and $\vec{b}$ $(\vec{a}+\vec{b}).(\vec{a}+\vec{b})=|\vec{a}|^2+|\vec{b}|^2$ if and only if $\vec{a}$ and $\vec{b}$ are orthogonal.
71. If $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}\neq\vec{0}$, show that $\vec{a}+\vec{c}=m\vec{b}$. Where m is a scalar.
72. If $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}\neq\vec{0}$, show that $\vec{b}=\vec{c}+t\vec{a}$. for some real number t.
73. For any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ show that $\vec{a}-\vec{b}$, $\vec{b}-\vec{c}$ and $\vec{c}-\vec{a}$ are coplanar.
74. Find the volume of parallelopiped whose continuous edges are represented by vectors $\vec{a}=2\hat{\imath}-3\hat{\jmath}+\hat{k}$, $\vec{b}=\hat{\imath}-\hat{\jmath}+2\hat{k}$, $\vec{c}=2\hat{\imath}+\hat{\jmath}-\hat{k}$
75. $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors suppose $\vec{a}.\vec{b}=\vec{a}.\vec{c}=0$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$. Prove that $\vec{a}=\pm 2(\vec{b}\times\vec{c})$.
76. For any two vectors $\vec{a}$ and $\vec{b}$, prove that $(\vec{a}\times\vec{b})^2=(\vec{a})^2(\vec{b})^2-(\vec{a}.\vec{b})^2$
77. Define vector product $\vec{a}\times\vec{b}$ of two vectors $\vec{a}$ and $\vec{b}$. If $|\vec{a}|=2$, $|\vec{b}|=5$, and $|\vec{a}\times\vec{b}|=8$. Find the value of $\vec{a}.\vec{b}$.
78. Define $\vec{a}\times\vec{b}$ and prove that $|\vec{a}\times\vec{b}|=(\vec{a}.\vec{b})\tan\theta$ where θ is angle between $\vec{a}$ and $\vec{b}$.
79. The volume of a parallelopiped, whose edges are $-12\hat{\imath}+\lambda\hat{k}$, $3\hat{\jmath}-\hat{k}$ and $2\hat{\imath}+\hat{\jmath}-15\hat{k}$ is 546 cubic units. Find the value of λ.
80. $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a}\times\vec{b}=\vec{c}$, $\vec{b}\times\vec{c}=\vec{a}$. Prove that $\vec{a}$, $\vec{b}$, $\vec{c}$ are mutually at right angles and $|\vec{b}|=1$, $|\vec{c}|=|\vec{a}|$.
81. If $\hat{a}$ and $\hat{b}$ are two unit vectors and θ is the angle between them, then show that: $\sin\frac{\theta}{2}=\frac{1}{2}|\hat{a}-\hat{b}|$.
82. If any three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar, show that the vectors $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are also coplanar.

Answers

1. $\frac{3}{7}\hat{\imath}-\frac{2}{7}\hat{\jmath}+\frac{6}{7}\hat{k}$
2. $\cos^{-1}\left(-\frac{1}{3}\right)$
3. $\frac{5}{2}$
4. $\frac{-2}{\sqrt{21}}\hat{\imath}+\frac{1}{\sqrt{21}}\hat{\jmath}+\frac{4}{\sqrt{21}}\hat{k}$
5. $30°$
6. $\frac{1}{3}\hat{\imath}+\frac{2}{3}\hat{\jmath}+\frac{2}{3}\hat{k}$
7. $\sqrt{3}$
8. $\frac{7}{\sqrt{5}}\hat{\imath}-\frac{14}{\sqrt{5}}\hat{\jmath}$
9. $\frac{8}{7}$
10. $p=\frac{2}{3}$
11. $60°$
12. $\frac{27}{2}$
13. $\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}}$
14. 9
15. $\frac{1}{2}$
16. -3
17. $\frac{\pi}{4}$
18. $\frac{\pi}{3}$
19. $\pm\frac{1}{\sqrt{3}}$
20. -4
21. $\frac{-2}{\sqrt{30}},\frac{1}{\sqrt{30}},\frac{-5}{\sqrt{30}}$
22. $3\hat{\imath}+2\hat{\jmath}+\hat{k}$
23. 5
24. 1
25. 2
26. 6 sq.units
28. $(-3,2,8)$
31. $\vec{a}$ *and* $\vec{b}$ *are parallel*
32. $\vec{c}+\vec{a}-\vec{b}$
33. $m=2$
34. $a=-5,b=4$
35. $\lambda=\pm 12$
36. $-\frac{5\sqrt{3}}{2},\frac{5}{2}$
37. $\frac{1}{3}$
38. $\vec{x}$ *and* $\vec{y}$ are parallel
39. $(6,11)$
40. $\left(5,\frac{14}{3},-6\right)$
41. $\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}}$
42. $\cos^{-1}\left(\frac{3}{7}\right)$
43. 4
44. $\frac{\pi}{2}$
45. $\frac{3}{2}$
46. -1
47. $\frac{\pi}{4}$
48. $\frac{\sqrt{3}}{2}\hat{\imath}+\frac{1}{2}\hat{\jmath}$
49. $\frac{\pi}{3}$
50. $3\hat{\imath}+2\hat{k}$
55. $\frac{1}{\sqrt{165}}(-10\hat{\imath}-7\hat{\jmath}+4\hat{k})$
58. 2
59. $\frac{5}{3}\hat{\imath}+\frac{2}{3}\hat{\jmath}+\frac{2}{3}\hat{k}$
63. $\frac{2}{3}\hat{\imath}-\frac{2}{3}\hat{\jmath}-\frac{1}{3}\hat{k}$
64. $\vec{a}\perp\vec{b}$
67. $\sqrt{5}$
74. 14 cubic units
77. 6
79. $-3,179$

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