NCERT EXAMPLER(XII)

Important Examples and Questions

RELATIONS AND FUNCTIONS

Questions

1. Let D be the domain of the real valued function f defined by $f(x) = \sqrt{25 - x^2}$. Then write D. $[Ans: [5, -5]]$
2. Let $f: R \to R$ be a function defined by $f(x) = 2x - 3, \forall\, x \in R$. Write f^{-1}. $\left[Ans: f^{-1}(x) = \frac{x+3}{2}\right]$
3. If $f: R \to R$ is defined by $f(x) = x^2 - 3x + 2$, write $f\{f(x)\}$. $[Ans: x^4 - 6x^3 + 10x^2 - 3x]$
4. Are the following sets of ordered pairs functions? If so examine whether the mapping is injective or surjective.
   (i) $\{(x, y): x\ is\ a\ person, y\ is\ the\ mother\ of\ x\}$ $[Ans: Not\ injective\ but\ surjective]$
   (ii) $\{(a, b): a\ is\ a\ person, b\ is\ an\ ancestor\ of\ a\}$ $[Ans: Not\ a\ function]$
5. If the mappings f and g are given by $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$. Write fog. $[Ans: \{(2, 5), (5, 2), (1, 5)\}]$
6. Let C be the set of complex numbers. Prove that the mapping $f: C \to R$ given by $f(z) = |z|, \forall\, z \in C$ is neither one-one nor onto.
7. If functions $f: A \to B$ and $g: B \to A$ satisfy $gof = I_A$, then show that f is one-one and g is onto.
8. Let $f: R \to R$ be the function defined by $f(x) = \frac{1}{2 - \cos x}, \forall\, x \in R$ then, find the range of f. $\left[Ans: \left[\frac{1}{3}, 1\right]\right]$
9. Let n be a fixed positive integer. Define a relation R in Z as follows $\forall\, a, b \in Z, aRb$ if and only if $a - b\ is\ divisible\ by\ n$. Show that R is an equivalence relation.
10. Let $A = R - \{3\}, B = R - \{1\}. If\ f: A \to B$ be defined by $f(x) = \frac{x-2}{x-3}, \forall\, x \in A$. Then show that f is bijective.

11. Let $A = \{1,2,3, \ldots, 9\}$ and R be the relation in $A \times A$ defined by $(a,b)R(c,d)$ if $a + d = b + c$ for $(a,b),(c,d)$ in $A \times A$. Prove that R is an equivalence relation and also obtain the equivalence class $[(2,5)]$.
12. Using the definition, prove that the function $f: A \to B$ is invertible if and only if f is both one-one and onto.
13. If $f: [2, \infty) \to R$ be the function defined by $f(x) = x^2 - 4x + 5$ then find the range of f. $[Ans: [1, \infty)]$.
14. If $f: N \to R$ be the function defined by $f(x) = \frac{2x-1}{2}$ and $g: Q \to R$ be another function defined by $g(x) = x + 2$. Then find $gof\left(\frac{3}{2}\right)$. $[Ans: 3]$.
15. If $f(x) = [4 - (x-7)^3]$, then find $f^{-1}(x)$. $\left[Ans: 7 + (4-x)^{\frac{1}{3}}\right]$.

Examples

1. For the set $A = \{1, 2, 3\}$ define a relation R in the set A as follows: $R = \{(1,1),(2,2),(3,3),(1,3)\}$ write the ordered pairs to be added to R to make it the smallest equivalence relation. $[Ans: (3,1)]$
2. Let R be the equivalence relation in the set Z of integers given by $R = \{(a,b): 2 \text{ divides } a - b\}$. Write the equivalence class $[0]$. $[Ans: \{0, \pm 2, \pm 4, \pm 6, \ldots\}]$
3. Show that the function $f: R \to R$ defined by $f(x) = \frac{x}{x^2+1}, \forall\, x \in R$ is neither one-one nor onto.
4. Let $f, g: R \to R$ be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x\ \forall\, x \in R$. Then find fog, gof
$\left[Ans: gof(x) = 0, \forall\, x \in R, fog(x) = \begin{cases} 0, & x > 0 \\ -4x, & x < 0 \end{cases}\right]$.
5. Set A has 3 elements and the set B has 4 elements. Then find the number of injective mappings that can be defined from A to B. $[Ans: 24]$
6. Find the domain of the function $f: R \to R$ defined by $f(x) = \sqrt{x^2 - 3x + 2}$.
$[Ans: (-\infty,] \cup [2, \infty)]$
7. Consider the set A containing n elements. Then find total number of injective functions from A onto itself. $[Ans: n!]$

INVERSE TRIGONOMETRIC FUNCTIONS

Questions

1. Find the value of $\tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{13\pi}{6}\right)$. $[Ans: 0]$
2. Prove that $\cot\left(\frac{\pi}{4} - 2\cot^{-1} 3\right) = 7$.

3. Show that $2\tan^{-1}(-3) = \frac{-\pi}{2} + \tan^{-1}\left(-\frac{4}{3}\right)$.
4. Find the real solution of $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$ $[Ans: 0, -1]$
5. Find the value of $\sin\left(2\tan^{-1}\frac{1}{3}\right) + \cos(\tan^{-1} 2\sqrt{2})$. $\left[Ans: \frac{14}{15}\right]$
6. Show that $\cos\left(2\tan^{-1}\frac{1}{7}\right) = \sin\left(4\tan^{-1}\frac{1}{3}\right)$.
7. Find the simplified form of $\cos^{-1}\left(\frac{3}{5}\cos x + \frac{4}{5}\sin x\right), where\ x \in \left[\frac{-3\pi}{4}, \frac{\pi}{4}\right]$

   $\left[Ans:\ \tan^{-1}\frac{4}{3} - x\right]$
8. Find the value of $4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{239}$ $\left[Ans: \frac{\pi}{4}\right]$
9. Show that $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$ and justify why the other value $\frac{4+\sqrt{7}}{3}$ is ignored?
10. If $a_1, a_2, a_3, \ldots, a_n$ is an arithmetic progression with common difference d, then evaluate the following expression.

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \tan^{-1}\left(\frac{d}{1+a_3a_4}\right) + \cdots + \tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right]$$

$\left[Ans: \frac{a_n - a_1}{1+a_1a_n}\right]$

11. If $3\tan^{-1}x + \cot^{-1}x = \pi$, then find x. $[Ans: 1]$
12. Find the value of $\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right]$. $\left[Ans: \frac{-\pi}{10}\right]$
13. Find the domain of the function $\cos^{-1}(2x-1)$. $[Ans: [0,1]]$
14. Find the domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ $[Ans: [1,2]]$
15. Find the value of $\sin[2\tan^{-1}(0.75)]$. $[Ans: 0.96]$.
16. If $\tan^{-1}x + \tan^{-1}y = \frac{4\pi}{5}$, then find the value of $\cot^{-1}x + \cot^{-1}y$. $\left[Ans: \frac{\pi}{5}\right]$
17. If $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$, then find the value of

    $\alpha(\beta+\gamma) + \beta(\gamma+\alpha) + \gamma(\alpha+\beta)$. $[Ans: 6]$.
18. The number of real solutions of the equation

    $\sqrt{1+\cos 2x} = \sqrt{2}\cos^{-1}(\cos x)\ in\ \left[\frac{\pi}{2}, \pi\right]$? $[Ans: no\ solution]$
19. If $\cos^{-1}x > \sin^{-1}x$, then find x $\left[Ans: \frac{1}{\sqrt{2}} < x \leq 1\right]$

Examples

1. Prove that $\tan(\cot^{-1}x) = \cot(\tan^{-1}x)$. State with reason whether the equality is valid for all values of x.
2. Prove that $\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18 = \cot^{-1}3$.
3. Which is greater, $\tan 1$ *or* $\tan^{-1}1$. $[Ans:\ \tan 1]$.
4. Find the values of x which satisfy the equation $\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x$

   $\left[Ans: 0, \frac{1}{2}\right]$

5. Solve the equation $\sin^{-1} 6x + \sin^{-1} 6\sqrt{3}\, x = -\frac{\pi}{2}$. $\left[Ans: -\frac{1}{12}\right]$
6. Show that $2\tan^{-1}\left\{\tan\frac{\alpha}{2}.\tan\left(\frac{\pi}{4}-\frac{\beta}{2}\right)\right\} = \tan^{-1}\frac{\sin\alpha.\cos\beta}{\cos\alpha+\sin\beta}$
7. Find the greatest and least values of $(\sin^{-1} x)^2 + (\cos^{-1} x)^2$ $\left[Ans: \frac{5\pi^2}{4} \text{ and } \frac{\pi^2}{8}\right]$
8. Find the domain of the function $y = \sin^{-1}(-x^2)$. $[Ans: -1 \le x \le 1]$.
9. Find the domain of $y = \cos^{-1}(x^2 - 4)$. $\left[Ans: [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]\right]$
10. Find the domain of the function defined by $f(x) = \sin^{-1} x + \cos x$. $[Ans: [-1, 1]]$
11. Find the value of $\sin(2\sin^{-1}(0.6))$. $[Ans: 0.96]$
12. If $\alpha \le 2\sin^{-1} x + \cos^{-1} x \le \beta$ then find α and β. $[Ans: \alpha = 0, \beta = \pi]$

MATRIX AND DETERMINANTS

Questions

1. Construct a 3×2 matrix whose elements are given by $a_{ij} = e^{ix} \sin jx$.

$$\left[Ans: \begin{bmatrix} e^x \sin x & e^x \sin 2x \\ e^{2x} \sin x & e^{2x} \sin 2x \\ e^{3x} \sin x & e^{3x} \sin 2x \end{bmatrix}\right]$$

2. Find the matrix A satisfying the matrix equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\left[Ans: \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\right]$$

3. Find A, if $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$ $\left[Ans: \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}\right]$
4. Solve for x and y, $x\begin{bmatrix} 2 \\ 1 \end{bmatrix} + y\begin{bmatrix} 3 \\ 5 \end{bmatrix} + \begin{bmatrix} -8 \\ -11 \end{bmatrix} = 0$. $[Ans: x = 1, y = 2]$
5. If $A = [3 \quad 5]$ and $B = [7 \quad 3]$, then find a non-zero matrix C such that $AC = BC$.
6. Show that $A^T A$ and AA^T both are symmetric matrices for any matrix A.
7. Let A and B be square matrices of the order 3×3. Is $(AB)^2 = A^2B^2$? Give reasons. $[Ans: True]$
8. If $A = \begin{bmatrix} \cos q & \sin q \\ -\sin q & \cos q \end{bmatrix}$, then show that $A^2 = \begin{bmatrix} \cos 2q & \sin 2q \\ -\sin 2q & \cos 2q \end{bmatrix}$
9. Prove by mathematical induction that $(A^T)^n = (A^n)^T$ where $n \in N$ for any square matrix A.
10. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ and $A^{-1} = A^T$, then find the value of α. $[Ans: \alpha \in R]$
11. If $P(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, show that $P(x).P(y) = P(x+y) = P(y).P(x)$

12. Find x, y and z if $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies $A^T = A^{-1}$.

$\left[Ans: x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{6}}, z = \pm\frac{1}{\sqrt{3}}\right]$

13. If $A = \frac{1}{\pi}\begin{bmatrix} \sin^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(x\pi) \end{bmatrix}$ and $B = \frac{1}{\pi}\begin{bmatrix} -\cos^{-1}((x\pi)) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(x\pi) \end{bmatrix}$, then prove that $A - B = \frac{1}{2}I$

14. If A is a skew-symmetric matrix, then prove that A^2 is a symmetric matrix.

15. Prove that: $\begin{vmatrix} y^2z^2 & yz & y+z \\ z^2x^2 & zx & z+x \\ x^2y^2 & xy & x+y \end{vmatrix} = 0.$

16. Prove that: $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$

17. If $A + B + C = 0$, then prove that $\begin{vmatrix} 1 & \cos C & \cos B \\ \cos C & 1 & \cos A \\ \cos B & \cos A & 1 \end{vmatrix} = 0.$

18. Find the value of θ satisfying $\begin{vmatrix} 1 & 1 & \sin 3\theta \\ -4 & 3 & \cos 2\theta \\ 7 & -7 & -2 \end{vmatrix} = 0.$

$\left[Ans: \theta = n\pi, \theta = n\pi + (-1)^n\frac{\pi}{6}\right]$

19. If $a_{1,}\ a_{2,}\ a_{3,} \ldots, a_r$ are in G.P. then prove that the determinant $\begin{vmatrix} a_{r+1} & a_{r+5} & a_{r+9} \\ a_{r+7} & a_{r+11} & a_{r+15} \\ a_{r+11} & a_{r+17} & a_{r+21} \end{vmatrix}$ is independent of r.

20. Show that the points $(a+5, a-4), (a-2, a+3)$ *and* (a, a) do not lie on a straight line for any value of a.

21. Show that ΔABC is an isosceles triangle, if the determinant

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1+\cos A & 1+\cos B & 1+\cos C \\ \cos^2 A + \cos A & \cos^2 B + \cos B & \cos^2 C + \cos C \end{vmatrix} = 0$$

22. If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find the value of A^{-1}. Using A^{-1}, solve the system of linear equations $x - 2y = 10, 2x - y - z = 8, -2y + z = 7$.

$[Ans: x = 0, y = -5, z = -3]$

23. If $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, the find BA and use this to solve the system of equations $y + 2z = 7, x - y = 3$ *and* $2x + 3y + 4z = 17$.

$[Ans: x = 2, y = -1, z = 4]$.

24. Prove that: $\begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2 \end{vmatrix}$ is divisible by $(a+b+c)$ and find the quotient. $\left[Ans: (a^3+b^3+c^3-3abc)[(a-b)^2+(b-c)^2+(c-a)^2]\right]$.

25. Find the number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$. $[Ans: 1]$

26. If $f(t) = \begin{bmatrix} \cos t & t & 1 \\ 2\sin t & t & 2t \\ \sin t & t & t \end{bmatrix}$, then find $\lim\limits_{t\to 0} \frac{f(t)}{t^2}$. $[Ans: 0]$

27. If x, y, z all are different from zero and $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = 0$. Then find the value of $x^{-1}+y^{-1}+z^{-1}$. $[Ans: -1]$

28. If there is two values of a which makes determinant $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$, then find the sum of these numbers. $[Ans: -4]$

29. If $x=-9$ is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$, then find other two roots. $[Ans: x=2, x=7]$

30. If $f(x) = \begin{vmatrix} (1+x)^{17} & (1+x)^{19} & (1+x)^{23} \\ (1+x)^{23} & (1+x)^{29} & (1+x)^{34} \\ (1+x)^{41} & (1+x)^{43} & (1+x)^{47} \end{vmatrix} = A + Bx + Cx^2 + \cdots$, then find A. $[Ans: 0]$

Examples

1. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$. Then show that $A^2 - 4A + I = O$. Using this result calculate A^5.

$\left[Ans: \begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}\right]$

CONTINUITY AND DIFFRENTIABILITY

Questions

1. Discuss the continuity of $f(x)$ where $f(x) = \begin{cases} \frac{1-\cos 2x}{x^2}, & if\ x \ne 0 \\ 5, & if\ x = 0 \end{cases}$ at $x = 0$.

$[Ans: Discontinous]$

2. Discuss the continuity of $f(x)$ where

$f(x) = \begin{cases} \frac{|x-4|}{2(x-4)}, & if\ x \ne 4 \\ 0, & if\ x = 4 \end{cases}$ at $x = 4$ $[Ans: Discontinous]$

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3. Discuss the continuity of $f(x)$ where

$f(x) = \begin{cases} |x|\cos\frac{1}{x}, if\ x \neq 0 \\ 0, \qquad if\ x = 0 \end{cases}$ at $x = 0$ $[Ans: continous]$

4. Discuss the continuity of $f(x)$ where $f(x) = \begin{cases} |x-a|\sin\frac{1}{x-a}, if\ x \neq 0 \\ 0, \qquad if\ x = a \end{cases}$ at $x = a$.

$[Ans: continous]$

5. Discuss the continuity of $f(x)$ where $f(x) = \begin{cases} \frac{e^{\frac{1}{x}}}{1+e^{\frac{1}{x}}}, if\ x \neq 0 \\ 0, \qquad if\ x = 0 \end{cases}$ at $x = 0$.

$[Ans: Discontinous]$

6. If $f(x)$, where $f(x) = \begin{cases} \frac{2^{x+2}-16}{4^x-16}, if\ x \neq 2 \\ k, \qquad if\ x = 2 \end{cases}$ is continuous at $x = 2$. Then find the value of k. $\left[Ans: k = \frac{1}{2}\right]$

7. If $f(x) = \begin{cases} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, if -1 \leq x < 0 \\ \frac{2x+1}{x-1}, \qquad if\ 0 \leq x \leq 1 \end{cases}$ is continuous at $x = 0$. Then find the value of k. $[Ans: -1]$

8. If $f(x) = \begin{cases} \frac{1-\cos kx}{x\sin x}, if\ x \neq 0 \\ \frac{1}{2}, \qquad if\ x = 0 \end{cases}$ is continuous at $x = 0$. Then find the value of k.

$[Ans: \pm 1]$

9. Prove that the function f defined by $f(x) = \begin{cases} \frac{x}{|x|+2x^2}, if\ x \neq 0 \\ k, \qquad if\ x = 0 \end{cases}$ remains discontinuous at $x = 0$, regardless the choice of k.

10. If the function $f(x) = \frac{1}{x+2}$, then find the points of discontinuity of the composite function $y = f\{f(x)\}$ $\left[Ans: x = -\frac{5}{2}\right]$

11. Find all the points of discontinuity of the function $f(t) = \frac{1}{t^2+t-2}$ where $t = \frac{1}{x-1}$.

12. Examine the differentiability of f, where f is defined by

$f(x) = \begin{cases} x[x], \qquad if\ 0 \leq x < 2 \\ (x-1)x, \qquad if\ 2 \leq x < 3 \end{cases}$ at $x = 2$. $[Ans: Not\ differentiable]$

13. Examine the differentiability of f, where f is defined by

$f(x) = \begin{cases} x^2\sin\frac{1}{x}, \quad if\ x \neq 0 \\ 0, \qquad if\ x = 0 \end{cases}$ at $x = 0$. $[Ans: differentiable]$

14. A function $f: R \to R$ satisfies the equation $f(x+y) = f(x).f(y)$ for all $x, y \in R$, $f(x) \neq 0$. Suppose that the function is differentiable at $x = 0$ and $f'(0) = 2$, then prove that $f(x) = 2f(x)$.

15. Differentiate w.r.t. x: $2^{\cos^2 x}$ $\left[Ans: -2^{\cos^2 x}\log 2\ (\sin 2x)\right]$

16. Differentiate w.r.t. x: $\log[\log(\log x^5)]$ $\left[Ans: \frac{5}{x.\log(\log x^5).(\log x^5)}\right]$
17. Differentiate w.r.t x: $\cos^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right)$, $-\frac{\pi}{4} < x < \frac{\pi}{4}$ $[Ans: -1]$
18. Find $\frac{dy}{dx}$: $x = e^{\theta}\left(\theta+\frac{1}{\theta}\right), y = e^{-\theta}\left(\theta-\frac{1}{\theta}\right)$. $\left[Ans: e^{-2\theta}\left(\frac{-\theta^3+\theta^2+\theta+1}{\theta^3+\theta^2+\theta-1}\right)\right]$
19. Find $\frac{dy}{dx}$: $x = 3\cos\theta - 2\cos^3\theta$, $y = 3\sin\theta - 2\sin^3\theta$. $[Ans: \cot\theta]$
20. Find $\frac{dy}{dx}$: $\sin x = \frac{2t}{1+t^2}, \tan y = \frac{2t}{1-t^2}$. $[Ans: 1]$
21. Find $\frac{dy}{dx}$: $x = \frac{1+\log t}{t^2}, y = \frac{3+2\log t}{t}$. $[Ans: t]$
22. If $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$, then prove that $\frac{dy}{dx} = -\frac{y\log x}{x\log y}$.
23. If $x = a\sin 2t(1+\cos 2t)$ and $y = b\cos 2t(1-\cos 2t)$, then show that $\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}} = \frac{b}{a}$.
24. Differentiate $\frac{x}{\sin x}$ w.r.t. $\sin x$. $\left[Ans: \frac{\tan x - x}{\sin^2 x}\right]$
25. Differentiate $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$ w.r.t. $\tan^{-1} x$ when $x \neq 0$. $\left[Ans: \frac{1}{2}\right]$
26. If $x = e^{\frac{x}{y}}$, then prove that $\frac{dy}{dx} = \frac{x-y}{x\log x}$.
27. If $y^x = e^{y-x}$, then prove that $\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$.
28. If $y = (\cos x)^{(\cos x)^{(\cos x)^{\cdots\infty}}}$, then show that $\frac{dy}{dx} = \frac{y^2\tan x}{y\log\cos x - 1}$
29. If $x\sin(a+y) + \sin a.\cos(a+y) = 0$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$
30. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$
31. Verify Rolle's theorem for the function $f(x) = \sin^4 x + \cos^4 x$ in $\left[0, \frac{\pi}{2}\right]$.
32. Verify Rolle's theorem for the function $f(x) = \log(x^2+2) - \log 3$ in $[-1, 1]$.
33. Verify Rolle's theorem for the function $f(x) = x(x+3)e^{\frac{-x}{2}}$ in $[-3, 0]$.
34. Discuss the applicability of Roll's theorem on the function given by
$f(x) = \begin{cases} x^2+1, if\ 0 \leq x \leq 1 \\ 3-x, \ \ if 1 \leq x \leq 2 \end{cases}$ $[Ans: Not\ applicable]$
35. Using Rolle's theorem find the point on the curve $y = (\cos x - 1)$ in $[0, 2\pi]$ where the tangent is parallel to X-axis. $[Ans: (\pi, -2)]$
36. Using Rolle's theorem, find the point on the curve $y = x(x-4), x \in [0, 4]$, where the tangent is parallel to X-axis. $[Ans: (2, -4)]$
37. Verify the mean value theorem for the function $f(x) = \frac{1}{4x-1}$ in $[1, 4]$
38. Verify the mean value theorem for the function $f(x) = \sin x - \sin 2x$ in $[0, \pi]$
39. Using Mean value theorem find a point on the curve $y = (x-3)^2$, where the tangent is parallel to the chord joining the points $(3, 0)$ and $(4, 1)$. $\left[Ans: \left(\frac{7}{2}, \frac{1}{4}\right)\right]$

40. Using mean value theorem, prove that there is a point on the curve $y = 2x^2 - 5x + 3$ between the points $A(1, 0)$ and $B(2, 1)$, where tangent is parallel to the chord AB. Also find the point $\left[Ans: \left(\frac{3}{2}, 0\right)\right]$

41. Find the value of p and q so that $f(x) = \begin{cases} x^2 + 3x + p, & if\ x \leq 1 \\ qx + 2, & if\ x > 1 \end{cases}$ is differentiable at $x = 1$. $[Ans: p = 3, q = 5]$.

42. If $x^m . y^n = (x + y)^{m+n}$, prove that (i) $\frac{dy}{dx} = \frac{y}{x}$ (ii) $\frac{d^2y}{dx^2} = 0$

43. If $x = \sin t$ and $y = \sin pt$, then prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2 y = 0$

44. Find the value of $\frac{dy}{dx}$, if $y = x^{\tan x} + \sqrt{\frac{x^2+1}{2}}$.

$\left[Ans: x^{\tan x}\left[\frac{\tan x}{x} + \log x . \sec^2 x\right] + \frac{x}{\sqrt{2(x^2+1)}}\right]$

Examples

1. If $e^x + e^y = e^{x+y}$, prove that $\frac{dy}{dx} = -e^{y-x}$

APPLICATION OF DERIVATIVES

Questions

1. A spherical ball of salt is dissolving in water in such a manner that the rate of decrease of the volume at any instant is proportional to the surface. Prove that the radius is decreasing at a constant rate.
2. A kite is moving horizontally at a height of 151.5m. If the speed of kite is 10 m/s, how fast is the string being let out, when the kite is 250m away from the boy who is flying the kite, if the height of the boy is 1.5m? $[Ans: 8\ m/s]$
3. Two men A and B start with velocity V at the same time from the junction of two roads inclined at 45° to each other. If they travel by different roads, then find the rate at which they are being separated. $\left[Ans: \sqrt{2 - \sqrt{2}}\right]$
4. Find an angle θ, where $0 < \theta < \frac{\pi}{2}$, which increases twice as fast as its sine. $\left[Ans: \frac{\pi}{3}\right]$
5. Find the approximate value of $(1.999)^5$. $[Ans: 31.920]$
6. A man, 2 m tall walk at the rate of $1\frac{2}{3}\ m/s$ towards a street light which is $5\frac{1}{3}\ m$ above the ground. At what rate is the tip of shadow moving and at what rate is the length of the shadow changing when he is $3\frac{1}{3}\ m$. From the base of the light?

$\left[Ans: 1m/s, 2\frac{2}{3}m/s\right]$

7. If x and y are the sides of two squares such that $y = x - x^2$, then find the rate of change of the area of second square with respect to the areas of first square. $[Ans: 2x^2 - 3x + 1]$
8. Find the condition that the curves $2x = y^2$ and $2xy = k$ intersect orthogonally. $[Ans: k^2 = 8]$
9. Prove that the curves $xy = 4$ *and* $x^2 + y^2 = 4$ touches each other.
10. Find the angle of intersection of the curves $y = 4 - x^2$ and $y = x^2$. $\left[Ans:\ \tan^{-1}\left(\frac{4\sqrt{2}}{7}\right)\right]$
11. Prove that the curves $y^2 = 4x$ and $x^2 + y^2 - 6x + 1 = 0$ touches each other at the point $(1, 2)$.
12. Show that $f(x) = 2x + \cot^{-1} x + \log\left(\sqrt{1 + x^2} - x\right)$ is increasing in R.
13. Show that for $a \geq 1$, $f(x) = \sqrt{3}\sin x - \cos x - 2ax + b$ is decreasing in R.
14. At what point, the slope of the curve $y = -x^3 + 3x^2 + 9x - 27$ is maximum? Also find the maximum slope. $[Ans: x = 1, 12]$
15. If the sum of lengths of the hypotenuse and a side of a right angled triangle is given, then show that the area of triangle is maximum, when the angle between them is $\frac{\pi}{3}$.
16. If an open box with square base is to be made of a given quantity of card board of area c^2 , then show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ $cu\ units$.
17. Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a volume as large as possible, when revolved about one of its side. Also find the maximum volume. $[Ans: 864\pi\ cm^3]$.
18. If the sum of the surface areas of cube and a sphere is constant, what is the ratio of an edge of the cube to the diameter of the sphere, when the sum of their volumes is minimum? $[Ans: 1:1]$
19. If AB is a diameter of a circle and C is any point on the circle, then show that the area of ΔABC is maximum, when it is isosceles.
20. A metal box with a square base and vertical sides is to contain 1024 cm^3. If the material for the top and bottom costs Rs.5 per cm^2 and the material for the sides cost Rs.2.50 per cm^2. Then find the least cost of the box. $[Ans: Rs. 1920]$
21. The sum of surface areas of a rectangular parallelopiped with sides $x, 2x$ *and* $\frac{x}{3}$ and a sphere is given to be constant. Prove that the sum of their volumes is minimum, if x is equal to three times the radius of the sphere. Also find the minimum value of the sum of their volumes. $\left[Ans:\ \frac{2}{3}x^3\left(1 + \frac{2\pi}{27}\right)\right]$
22. If the sides of an equilateral triangle are increasing at the rate of 2 cm/s then find the rate at which the area increases, when the side is 10cm. $[Ans: 10\sqrt{3}cm^2/s]$

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23. A ladder, 5m long standing on a horizontal floor, leans against a vertical wall. If the top of the ladder slides downwards at the rate of 10 cm/s, then find the rate at which the angle between the floor and the ladder is decreasing when lower end of ladder is 2m from the wall. $\left[Ans: \frac{1}{20} rad/s\right]$
24. If the curve $ay + x^2 = 7$ and $x^3 = y$, cut orthogonally at $(1, 1)$, then find the value of a . $[Ans: a = 6]$
25. If $y = x^4 - 10$ and x changes from 2 to 1.99, then what is the change in y? $[Ans: 0.32]$
26. Find the equation of tangent to the curve $y(1 + x^2) = 2 - x$ where it crosses X-axis. $[Ans: 5y + x = 2]$
27. Find the point where the tangent to the curve $y = e^{2x}$ at the point $(0, 1)$ meet X-axis. $\left[Ans: (-\frac{1}{2}, 0)\right]$
28. Find the slope of the tangent to the curve $x = t^2 + 3t - 8$ and $y = 2t^2 - 2t - 5$ at the point $(2, -1)$. $\left[Ans: \frac{6}{7}\right]$
29. Find the angle of intersection of two curves $x^3 - 3xy^2 + 2 = 0$ *and* $3x^2y - y^3 - 2 = 0$. $\left[Ans: \frac{\pi}{2}\right]$
30. Prove that the function $f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$ is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.
31. Find the maximum value of $\sin x . \cos x$ $\left[Ans: \frac{1}{2}\right]$
32. Find the stationary point of the function $f(x) = x^x$. $\left[Ans: \frac{1}{e}\right]$
33. Find the maximum value of $\left(\frac{1}{x}\right)^x$. $\left[Ans: (e)^{\frac{1}{e}}\right]$
34. Find the values of a for which the function $f(x) = \sin x - ax + b$ is increasing on R. $[Ans: (-\infty, -1)]$
35. Find the interval in which the function $f(x) = \frac{2x^2-1}{x^4}$, $(where, x > 0)$ is decreasing. $[Ans: (1, \infty)]$
36. Find the least value of the function $f(x) = ax + \frac{b}{x}$ $(where, a > 0, b > 0, x > 0)$. $[Ans: 2\sqrt{ab}]$

Examples

1. For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/sec, then how fast is the slope of the curve changing when $x = 3$? $[Ans: 72\ units/sec]$
2. Water is dripping out from a conical funnel of semi-vertical angle $\frac{\pi}{4}$ at the uniform rate of $2\ cm^2/sec$ in the surface area through a tiny hole at the vertex of the bottom.

When the slant height of cone is 4cm, find the rate of decrease of the slant height of water. $\left[Ans: \frac{\sqrt{2}}{4\pi} cm/s\right]$

3. Find the angle of intersection of the curves $y^2 = x$ and $x^2 = y$. $\left[Ans: \frac{\pi}{2}, \tan^{-1}\left(\frac{3}{4}\right)\right]$
4. Prove that the function $f(x) = \tan x - 4x$ is strictly decreasing on $\left(\frac{-\pi}{3}, \frac{\pi}{3}\right)$.
5. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ has neither maxima nor minima.
6. Using differentials find the approximate value of $\sqrt{0.082}$. $[Ans: 0.2867]$
7. Find the condition for the curves $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1\, ;\, xy = c^2$ to intersect orthogonally. $[Ans: a^2 - b^2 = 0]$
8. Show that the local maximum value of $x + \frac{1}{x}$ is less than local minimum value.
9. Water is dripping out at a steady rate of 1cu cm/sec through a tiny hole at the vertex of conical vessel, whose axis is vertical. When the slant height of water in the vessel is 4cm, find the rate of decrease of slant height, where the vertical angle of the conical vessel is $\frac{\pi}{6}$. $\left[Ans: \frac{1}{2\sqrt{3}\pi}\ cm/s\right]$
10. Find the angle of intersection of the curves $y^2 = 4ax$ and $x^2 = 4by$.

$$\left[Ans: \frac{\pi}{2}, \tan^{-1}\left(\frac{3a^{\frac{1}{3}}.b^{\frac{1}{3}}}{2\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}\right)\right]$$

11. Show that the equation of normal at any point on the curve $x = 3\cos\theta - \cos^3\theta$, $y = 3\sin\theta - \sin^3\theta$ is $4(y\cos^3\theta - x\sin^3\theta) = 3\sin 4\theta$.
12. Find the maximum and minimum values of $f(x) = \sec x + \log\cos^2 x, 0 < x < 2\pi$. $[Ans: 1, -1,2(1 - \log 2)]$
13. Find the difference between the greatest and least values of the function $f(x) = \sin 2x - x$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. $[Ans: \pi]$
14. An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius a. Show that area of triangle is maximum when $\theta = \frac{\pi}{6}$.
15. Find the abscissa of the point on the curve $3y = 6x - 5x^3$, the normal at which passes through origin. $[Ans: 1]$
16. Find the angle formed by the tangent to the curve $x = e^t\cos t\ and\ y = e^t\sin t\ at\ t = \frac{\pi}{4}$ makes with X-axis. $\left[Ans: \frac{\pi}{2}\right]$
17. Find the point on the curve $y^2 = x$, where tangent makes an angle of $\frac{\pi}{4}$ with X-axis. $\left[Ans: \left(\frac{1}{4}, \frac{1}{2}\right)\right]$
18. Find the values of a for which $y = x^2 + ax + 25$ touches the axis of X. $[Ans: a = \pm 10]$

19. Find the maximum value of $f(x) = \frac{1}{4x^2+2x+1}$. $\left[Ans:\ \frac{4}{3}\right]$
20. Find the rate of change of volume of sphere with respect to its surface area, when the radius is 2cm. $[Ans: 1\ cm^3/cm^2]$

INTEGRALS

Questions

1. Evaluate: $\int \frac{x}{\sqrt{x}+1} dx$ $\left[Ans: 2\left[\frac{x\sqrt{x}}{3} - \frac{x}{2} + \sqrt{x} - \log\left|\left(\sqrt{x}+1\right)\right|\right] + c\right]$
2. Evaluate: $\int \frac{x^{\frac{1}{2}}}{1+x^{\frac{3}{4}}} dx$ $\left[Ans: \frac{4}{3}x^{\frac{3}{4}} - \log\left|\left(1+x^{\frac{3}{4}}\right)\right| + c\right]$
3. Evaluate: $\int \frac{\sqrt{1+x^2}}{x^4} dx$ $\left[Ans: -\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{\frac{3}{2}} + c\right]$
4. Evaluate: $\int \frac{x^2}{1-x^4} dx$ $\left[Ans: \frac{1}{4}\log\left|\frac{1+x}{1-x}\right| - \frac{1}{2}\tan^{-1}x + c\right]$
5. Evaluate: $\int \frac{1}{x\sqrt{x^4-1}} dx$ $\left[Ans: \frac{1}{2}\sec^{-1}(x^2) + c\right]$
6. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\tan x}{1+m^2\tan^2 x} dx$ $\left[Ans: \log\frac{m}{m^2-1}\right]$
7. Evaluate: $\int_0^{\pi} x \sin x \cos^2 x\ \ dx$ $\left[Ans: \frac{\pi}{3}\right]$
8. Evaluate: $\int_0^{\frac{1}{2}} \frac{1}{(1+x^2)\sqrt{1-x^2}} dx$ $\left[Ans: \frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)\right]$
9. Evaluate: $\int \frac{x^2}{x^4-x^2-12} dx$ $\left[Ans: \frac{1}{7}\log\left|\frac{x-2}{x+2}\right| + \frac{\sqrt{3}}{7}\tan^{-1}\frac{x}{\sqrt{3}} + c\right]$
10. Evaluate: $\int \frac{x^2}{(x^2+a^2)\,(x^2+b^2)} dx$ $\left[Ans: \frac{1}{a^2-b^2}\left[a\tan^{-1}\frac{x}{a} - b\tan^{-1}\frac{x}{b}\right] + c\right]$
11. Evaluate: $\int e^{\tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right) dx$ $\left[Ans: xe^{\tan^{-1}x} + c\right]$
12. Evaluate: $\int \sin^{-1}\sqrt{\frac{x}{a+x}} dx$ $\left[Ans: a\left[\frac{x}{a}\tan^{-1}\sqrt{\frac{x}{a}} + \tan^{-1}\sqrt{\frac{x}{a}}\right] + c\right]$
13. Evaluate: $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{\frac{5}{2}}} dx$ $\left[Ans: \frac{3}{2}\right]$
14. Evaluate: $\int e^{-3x}\cos^3 x\, dx$ $\left[Ans: \frac{e^{-3x}}{24}[\sin 3x - \cos 3x] + \frac{3e^{-3x}}{40}[\sin x - 3\cos x] + c\right]$
15. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{dx}{(a^2\cos^2 x + b^2\sin^2 x)^2}$ $\left[Ans: \frac{\pi}{4}\left(\frac{a^2+b^2}{a^3.b^3}\right)\right]$
16. Evaluate: $\int_0^1 x\log(1+2x)$ $\left[Ans: \frac{3}{8}\log 3\right]$
17. Evaluate: $\int_0^{\pi} x\log\sin x\, dx$ $\left[Ans: -\frac{\pi^2}{2}\log 2\right]$
18. Evaluate: $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \log(\sin x + \cos x)dx$ $\left[Ans: -\frac{\pi}{8}\log 2\right]$

19. Evaluate: $\int \frac{x^9}{(4x^2+1)^6} dx$ $\left[Ans: \frac{1}{10}\left(4+\frac{1}{x^2}\right)^{-5}+c\right]$
20. Evaluate: $\int \frac{x+\sin x}{1+\cos x} dx$ $\left[Ans: x\tan\frac{x}{2}+c\right]$
21. Evaluate: $\int_0^{\frac{\pi}{2}} \sqrt{1-\sin 2x}\ dx$ $[Ans: 2(\sqrt{2}-1)]$

Examples

1. Evaluate: $\int \tan^8 x \,.\sec^4 x\ dx$ $\left[Ans: \frac{\tan^{11} x}{11}+\frac{\tan^9 x}{9}+c\right]$
2. Show that $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\cos x+\sin x} dx = \frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$
3. Find: $\int_0^1 x(\tan^{-1} x)^2\, dx$ $\left[Ans: \frac{\pi^2-4\pi}{16}+\log\sqrt{2}\right]$
4. Evaluate: $\int_{-1}^{2} f(x)\,dx$, where $f(x)=|x+1|+|x|+|x-1|$ $\left[Ans: \frac{19}{2}\right]$
5. Evaluate: $\int_{-1}^{1} \frac{x^3+|x|+1}{x^2+2|x|+1} dx$ $[Ans: 2\log 2]$
6. Evaluate: $\int_{-2}^{2} |x\cos \pi x|\, dx$ $\left[Ans: \frac{8}{\pi}\right]$

APPLICATION OF INTEGRALS

Questions

1. Find the area of the region bounded by the parabola $y^2=2px$ and $x^2=2py$
$\left[Ans: \frac{4p^2}{3} sq\ units\right]$
2. Find the area of the region bounded by the curve $y=x^3$, $y=x+6$ and $x=0$.
$[Ans: 10 sq\ units]$
3. Calculate the area under the curve $y=2\sqrt{x}$ included between the lines
$x=0\ and\ x=1.$ $\left[Ans: \frac{4}{3} sq\ units\right]$
4. Find the area enclosed by the curve $y=-x^2$ and the straight line $x+y+2=0$.
$\left[Ans: \frac{9}{2} sq\ units\right]$
5. Find the area bounded by the curve $y=\sqrt{x}, x=2y+3$ in first quadrant and X-axis.
$[Ans: 9 sq\ units]$
6. Compute the area bounded by the lines $x+2y=2, y-x=1\ and\ 2x+y=7$.
$[Ans: 6 sq\ units]$
7. Find the area bounded by the lines $y=4x+5, y=5-x\ and\ 4y=x+5$
$\left[Ans: \frac{15}{2} sq\ units\right]$

Examples

1. Find the area of the region bounded by the parabola $y^2 = 2x$ and the straight line $x - y = 4$. [Ans: $18 sq\ units$]
2. Find the area enclosed by the curve $x = 3\cos t$ and $y = 2\sin t$. [Ans: $6\pi sq\ units$]
3. Find the area of the region included between the parabola $y = \frac{3x^2}{4}$ and the line $3x - 2y + 12 = 0$. [Ans: $27 sq\ units$]
4. Find the area of the region bounded by the curves $x = at^2$ and $y = 2at$ between the ordinate corresponding to $t = 1$ and $t = 2$. $\left[Ans: \frac{56}{3}a^2 sq\ units\right]$
5. Find the area of the region above the X-axis, included between the parabola $y^2 = ax$ and the circle $x^2 + y^2 = 2ax$.
6. Find the area of the minor segment of the circle $x^2 + y^2 = a^2$ and cut off by the line $x = \frac{a}{2}$. $\left[Ans: \frac{a^2}{12}(4\pi - 3\sqrt{3}) sq\ units\right]$

DIFFERENTIAL EQUATIONS

Questions

1. Find the solution of $\frac{dy}{dx} = 2^{y-x}$ [Ans: $2^{-x} - 2^{-y} = K$]
2. Find the differential equation of all non-vertical lines in a plane. $\left[Ans: \frac{d^2y}{dx^2} = 0\right]$
3. Solve $(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{1}{x^2-1}$ $\left[Ans: y.(x^2 - 1) = \frac{1}{2}\log\left(\frac{x-1}{x+1}\right) + K\right]$
4. Find general solution of $\frac{dy}{dx} + ay = e^{mx}$ [Ans: $(m + a)y = e^{mx} + Ke^{-ax}$]
5. Solve the differential equation $\frac{dy}{dx} + 1 = e^{x+y}$. [$Ans$: $(x + C)e^{x+y} + 1 = 0$]
6. Solve the differential equation $\frac{dy}{dx} = 1 + x + y^2 + xy^2$, when $y = 0$ and $x = 0$. $\left[Ans: y = \tan\left(x + \frac{x^2}{2}\right)\right]$
7. Find the general solution of $(x + 2y^3)\frac{dy}{dx} = y$ [Ans: $x = y^3 + Cy$]
8. If $y(x)$ is a solution of $\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx} = -\cos x$ and $y(0) = 1$ then find the value of $y\left(\frac{\pi}{2}\right)$. $\left[Ans: \frac{1}{3}\right]$
9. If $y(t)$ is a solution of $(1 + t)\frac{dy}{dt} - ty = 1$ and $y(0) = -1$ then show that $y(1) = -\frac{1}{2}$
10. Form the differential equation having $y = (\sin^{-1} x)^2 + A\cos^{-1} x + B$ where A and B are arbitrary constant as its general solution. $\left[Ans: (1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0\right]$

11. Form the differential equation of all circles which passes through origin and whose centre lie on y-axis. $\left[Ans: (x^2 - y^2)\frac{dy}{dx} - 2xy = 0\right]$
12. Find the equation of a curve passing through origin and satisfying the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$. $\left[Ans: y = \frac{4x^3}{3(1+x^2)}\right]$
13. Solve $(x + y)(dx - dy) = dx + dy$ $\quad [Ans: x + y = Ke^{x-y}]$
14. Find the differential equation of system of concentric circle with centre $(1, 2)$. $\left[Ans: (y - 2)\frac{dy}{dx} + (x - 1) = 0\right]$
15. Solve : $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$ $\left[Ans: y = -\cos x + \frac{2\sin x}{x} + \frac{2\cos x}{x^2} + \frac{x}{3}\log x - \frac{x}{9} + Cx^{-2}\right]$
16. Find the general solution of $(1 + \tan y)(dx - dy) + 2xdy = 0$. $[Ans: x(\sin y + \cos y) = \sin y + Ce^{-y}]$
17. Solve $\frac{dy}{dx} = \cos(x + y) + \sin(x + y)$ $\left[Ans:\ \log\left|1 + \tan\frac{(x+y)}{2}\right| = x + C\right]$
18. Find the equation of a curve passing through origin, if the slope of the tangent to the curve at any point (x, y) is equal to the square of the difference of the abscissa and ordinate of the point. $[Ans: (1 - x + y)e^{2x} = 1 + x - y]$
19. Find the equation of the curve passing through the point $(1, 1)$ if the tangent drawn at any point $P(x, y)$ on the curve meets the coordinate axes at A and B such that P is mid point of AB. $[Ans: xy = 1]$
20. Solve: $x\frac{dy}{dx} = y(\log y - \log x + 1)$. $\left[Ans:\ \log\left(\frac{y}{x}\right) = Cx\right]$
21. Find the degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ $\quad [Ans: 2]$
22. Find the order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} + x^{\frac{1}{5}} = 0$ $[Ans: 2, 4]$
23. Find the integrating factor of differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$. $\left[Ans:\ \frac{e^x}{x}\right]$

Examples

1. Find the equation of family of lines passing through the origin. $\left[Ans: x\frac{dy}{dx} - y = 0\right]$
2. Find the differential equation of all non-horizontal lines in a plane. $\left[Ans: \frac{d^2y}{dx^2} = 0\right]$
3. Find the equation of the curve passing through the point $(1, 1)$ if the perpendicular distance of the origin from the normal at any point $P(x, y)$ of the curve is equal to the distance of P from the X-axis. $[Ans: x^2 + y^2 - 2x = 0]$

VECTOR ALGEBRA

Questions

1. If $\vec{a}$ *and* $\vec{b}$ are the position vector of A and B respectively, then find position vector of a point C in BA produce such that $\overrightarrow{BC} = 1.5\overrightarrow{BA}$. $\left[Ans: \frac{3\vec{a}-\vec{b}}{2}\right]$
2. Using vectors, find the value of k, such that the points $(k, -10, 3), (1, -1, 3)$ and $(3, 5, 3)$ are collinear. $[Ans: k = -2]$
3. A vector $\vec{r}$ is inclined at equal angles to the three axes. If the magnitude of $\vec{r}$ is $2\sqrt{3}$ units, then find the value of $\vec{r}$. $\left[Ans: \pm 2\left(\hat{\imath} + \hat{\jmath} + \hat{k}\right)\right]$
4. If vector $\vec{r}$ has magnitude 14 and direction ratios 2, 3 and -6. Then find the direction cosines and components of $\vec{r}$, given that $\vec{r}$ makes an acute angle with X-axis. $\left[Ans: 4\hat{\imath} + 6\hat{\jmath} - 12\hat{k}\right]$
5. Find a vector of magnitude 6, which is perpendicular to both the vectors $2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $4\hat{\imath} - \hat{\jmath} + 3\hat{k}$. $[Ans: -2\hat{\imath} + 4\hat{\jmath} + 4\hat{k}]$
6. If $\vec{a} + \vec{b} + \vec{c} = 0$, then show that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$. Interpret the result geometrically.
7. If A, B, C and D are the points with position vectors $\hat{\imath} - \hat{\jmath} + \hat{k}, 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ *and* $3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ respectively, then find the projection of $\overrightarrow{AB}$ along $\overrightarrow{CD}$. $[Ans: \sqrt{21}\ units]$
8. Using vectors, prove that the parallelogram on the same base and between the same parallels are equal in area.
9. Prove that in any ΔABC, $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$, where a, b *and* c are the magnitudes of the sides opposite to the vertices A, B and C respectively.
10. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ determine the vertices of a triangle, show that $\frac{1}{2}[\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}]$ gives the vector area of the triangle. Hence deduce the condition that the three points $\vec{a}, \vec{b}$ *and* $\vec{c}$ are collinear. Also find the unit vector normal to the plane of the triangle. $\left[Ans: \frac{\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}}{|\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}|}\right]$
11. Show that area of the parallelogram whose diagonals are given by $\vec{a}$ *and* $\vec{b}$ is $\frac{|\vec{a} \times \vec{b}|}{2}$. Also find the area of the parallelogram, whose diagonals are $2\hat{\imath} - \hat{\jmath} + \hat{k}$ *and* $\hat{\imath} + 3\hat{\jmath} - \hat{k}$. $\left[Ans: \frac{1}{2}\sqrt{62}\ units\right]$
12. If $\vec{a} = \hat{\imath} - \hat{\jmath} + \hat{k}$ *and* $\vec{b} = \hat{\jmath} - \hat{k}$, then find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ *and* $\vec{a} \cdot \vec{c} = 3$. $\left[Ans: \frac{1}{3}(5\hat{\imath} + \hat{\jmath} + 2\hat{k})\right]$

13. Find the value of λ for which the vectors $3\hat{\imath} - 6\hat{\jmath} + \hat{k}$ and $2\hat{\imath} - 4\hat{\jmath} + \lambda\hat{k}$ are parallel. $\left[Ans: \lambda = \frac{2}{3}\right]$
14. For any vector $\vec{a}$, find the value of $(\vec{a} \times \hat{\imath})^2 + (\vec{a} \times \hat{\jmath})^2 + \left(\vec{a} \times \hat{k}\right)^2$. $[Ans: 2\vec{a}^2]$
15. If $|\vec{a}| = 10, |\vec{b}| = 2$ and $\vec{a}\,.\vec{b} = 12$, then find the value of $|\vec{a} \times \vec{b}|$. $[Ans: 16]$
16. If vectors $\lambda\hat{\imath} + \hat{\jmath} + 2\hat{k},\ \hat{\imath} + \lambda\hat{\jmath} - \hat{k}$ and $2\hat{\imath} - \hat{\jmath} + \lambda\hat{k}$ are coplanar, then find the value of λ. $[Ans: -2, 1 \pm \sqrt{3}]$
17. If $|\vec{a}| = 4$ and $-3 \le \lambda \le 2$, then find the range of $|\lambda\vec{a}|$. $[Ans: [0, 12]]$
18. Find the number of vectors of unit length perpendicular to the vectors $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\jmath} + \hat{k}$, $[Ans: \vec{c} = \pm(\vec{a} \times \vec{b})]$
19. If vector $\vec{a} + \vec{b}$ bisects the angle between the non-collinear vectors $\vec{a}$ and $\vec{b}$, then prove that $\vec{a}$ and $\vec{b}$ are equal vectors.
20. If $\vec{r}.\vec{a} = 0, \vec{r}.\vec{b} = 0$, and $\vec{r}.\vec{c} = 0$ for some non-zero vector $\vec{r}$, then find the value of $\vec{a}.(\vec{b} \times \vec{c})$. $[Ans: 0]$
21. If vectors $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ and $\vec{b} = -\hat{\imath} - 2\hat{k}$, are the adjacent sides of a parallelogram. Find the angle between its diagonals. $\left[Ans: \frac{\pi}{4}\right]$
22. Find the value of the expression $|\vec{a} \times \vec{b}|^2 + (\vec{a}\,.\vec{b})^2$. $\left[Ans: |\vec{a}|^2|\vec{b}|^2\right]$
23. If $|\vec{a} \times \vec{b}|^2 + |\vec{a}\,\vec{b}|^2 = 144$ and $|\vec{a}| = 4$, then find $|\vec{b}|$
24. If $\vec{a}$ is any non-zero vector, then find the value $(\vec{a}.\hat{\imath}).\hat{\imath} + (\vec{a}.\hat{\jmath}).\hat{\jmath} + (\vec{a}.\hat{k}).\hat{k}$.

THREE DIMENSIONAL GEOMETRY

Questions

1. Show that lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Also, find their point of intersection. $[Ans: (-1, -1, -1)]$
2. Prove that the line through $A(0, -1, -1)$ and $B(4, 5, 1)$ intersects the line through $C(3, 9, 4)$ and $D(-4, 4, 4)$.
3. Prove that the lines $x = py + q, z = ry + s$ and $x = p'y + q', z = r'y + s'$ are perpendicular, if $pp' + rr' + 1 = 0$.
4. Find the equation of a plane which bisects perpendicularly the line joining the points $A(2, 3, 4)$ and $B(4, 5, 8)$ at right angles. $[Ans: x + y + z = 9]$
5. If the line drawn from the point $(-2, -1, -3)$ meet a plane at right angle at the point $(1, -3, 3)$ then find the equation of the plane. $[Ans: 3x - 2y + 6z - 27 = 0]$
6. Find the equation of a plane which is at a distance $3\sqrt{3}$ units from the origin and the normal to which is equally inclined to the coordinate axis. $[Ans: x + y + z = 9]$

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7. Find the equations of two lines through the origin which intersect the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at angles of $\frac{\pi}{3}$ each. $\left[Ans: \frac{x}{1}=\frac{y}{2}=\frac{z}{-1}, \frac{x}{-1}=\frac{y}{1}=\frac{z}{-2}\right]$
8. Find the angle between the lines whose direction cosines are given by the equation $l+m+n=0$ and $l^2+m^2-n^2=0$. $\left[Ans: \frac{\pi}{3}\right]$
9. If a variable line in two adjacent position has direction cosines l, m, n and $l+\delta l, m+\delta m, n+\delta n$ then show that the small angle $\delta\theta$ between two positions is given by $\delta\theta^2=\delta l^2+\delta m^2+\delta n^2$
10. If O is the origin and A is (a, b, c) then find the direction cosines of the line OA and equation of the plane through A at right angle to OA.
$[Ans: ax+by+cz=a^2+b^2+c^2]$
11. Two systems of rectangular axis have the same origin. If a plane cuts them at a distances a, b, c and a', b', c' respectively from the origin,
then prove that $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}$
12. Find the foot of perpendicular from the point $(2, 3, -8)$ to the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$.
Also find the perpendicular distance from the given point to the line.
$\left[Ans: (2, 3, -8), 3\sqrt{5}\right]$
13. Find the distance of a point $(2, 4, -1)$ from the line $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$. $[Ans: 7\ units]$
14. Find the equation of the line passing through the point $(3, 0, 1)$ and parallel to the planes $x+2y=0$ and $3y-z=0$.
$[Ans: (x-3)\hat{\imath}+y\hat{\jmath}+(z-1)\hat{k}+\lambda(-2\hat{\imath}+\hat{\jmath}+3\hat{k})]$
15. Find the equation of the plane through the points $(2, 1, -1), (-1, 3, 4)$ and perpendicular to the plane $x-2y+4z=10$. $[Ans: 18x+17y+4z=49]$
16. If the plane $ax+by=0$ is rotated about its line of intersection with the plane $z=0$ through an angle α, then prove that the equation of the plane in its new position is
$$ax+by\pm\left(\sqrt{a^2+b^2}\ \tan\alpha\right)z=0$$
17. Find the equation of the plane through the intersection of the planes $\vec{r}.(\hat{\imath}+3\hat{\jmath})-6=0$ and $\vec{r}.(3\hat{\imath}-\hat{\jmath}-4\hat{k})=0$, whose perpendicular distance from origin is unity. $[Ans: -2x+4y+4z-6=0]$
18. Show that the points $(\hat{\imath}-\hat{\jmath}+3\hat{k})$ and $3(\hat{\imath}+\hat{\jmath}+\hat{k})$ are equidistant from the plane $\vec{r}.(5\hat{\imath}+2\hat{\jmath}-7\hat{k})+9=0$ and lies on opposite side of it.
19. $\overrightarrow{AB}=3\hat{\imath}-\hat{\jmath}+\hat{k}$ and $\overrightarrow{CD}=-3\hat{\imath}+2\hat{\jmath}+4\hat{k}$ are two vectors. The position vectors of the points A and C are $6\hat{\imath}+7\hat{\jmath}+4\hat{k}$ and $-9\hat{\imath}+2\hat{k}$ respectively. Find the position vector of a point P on the line AB and a point Q on the line CD such that $\overrightarrow{PQ}$ is perpendicular to $\overrightarrow{AB}$ and $\overrightarrow{CD}$ both.

20. Show that the straight lines whose direction cosines are given by $2l + 2m - n = 0$ and $mn + nl + lm = 0$ are at right angles.
21. If l_1, m_1, n_1 , l_2, m_2, n_2 and l_3, m_3, n_3 are the direction cosines of three mutually perpendicular lines, then prove that the line whose direction cosines are proportional to $l_1 + l_2 + l_3, m_1 + m_2 + m_3$ and $n_1 + n_2 + n_3$ makes equal angles with them.
22. Find the distance of the point (α, β, γ) from Y-axis. $\left[Ans: \sqrt{\alpha^2 + \gamma^2}\right]$
23. Find the reflection of the point (α, β, γ) in XY- plane. $[Ans: (\alpha, \beta, -\gamma)]$
24. Find the area of the quadrilateral ABCD where $A(0,4,1), B(2,3,-1), C(4,5,0)\ and\ D(2,6,2)$ $[Ans: 9\ sq.units]$
25. Find the locus represented by $xy + yz = 0$. $[Ans: a\ pair\ of\ perpendicular\ planes]$
26. If the plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1}\alpha$ with X-axis, then find the value of α. $\left[Ans: \frac{2}{7}\right]$
27. If the plane passes through the points $(2,0,0)$, $(0,3,0)$ and $(0,0,4)$ then find equation of the plane. $\left[Ans: \frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1\right]$

Examples

1. The $x - coordinate$ of a point on the line joining the points $Q(2,2,1)$ and $R(5,1,-2)$ is 4. Find its z-coordinate. $[Ans: -1]$
2. Find the distance of the point $(-2,4,-5)$ from the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$. $\left[Ans: \sqrt{\frac{37}{10}}\right]$
3. Find the coordinates of the point where the line through $(3,-4,-5)\ and\ (2,-3,1)$ crosses the plane passing through three points $(2,2,1), (3,0,1)\ and\ (4,-1,0)$. $[Ans: (1,-2,7)]$
4. A plane meets the coordinate axis in A, B, C such that the centroid of ΔABC is the point (α, β, γ). Show that the equation of the plane is $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$.
5. Find the angle between the lines whose direction cosines are given by the equations: $3l + m + 5n = 0\ and\ 6mn - 2nl + 5lm = 0$. $\left[Ans: \cos^{-1}\left(-\frac{1}{6}\right)\right]$
6. Find the coordinates of the foot of perpendicular drawn from the point A$(1,8,4)$ to the line joining the points $B(0,-1,3)\ and\ C(2,-3,-1)$. $\left[Ans: \left(-\frac{5}{3}, \frac{2}{3}, \frac{19}{3}\right)\right]$
7. Find the image of the point $(1,6,3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. $[Ans: (1,0,7)]$
8. Find the coordinates of the foot of perpendicular drawn from the point $(2,5,7)$ on the X-axis. $[Ans: (2,0,0)]$
9. Find the distance of point $P(a,b,c)$ from X-axis. $\left[Ans: \sqrt{b^2 - c^2}\right]$

10. If a line makes angle α, β, γ with positive direction of coordinate axes then find the value of $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$. $[Ans: 2]$

LINEAR PROGRAMMING

Questions

1. A manufacturer of electronic circuits has a stock of 200 resistors, 120 transistors and 150 capacitors and is required to produce two types of circuits A and B. Type A requires 20 resistors, 10 transistors and 10 capacitors. Type B require 10 resistors, transistors and 30 capacitors. If the profit on type A circuit is Rs.50 and that on type B is Rs.60, formulate this problem as LPP, so that the manufacturer can maximize his profit. Find the maximum profit. $[Ans: Rs. 480]$
2. A firm has to transport 1200 packages using large vans which can carry 200 packages each and small van which can take 80 packages each. The cost for engaging each large van is Rs.400 and each small van is Rs.200. Not more than Rs.3000 is to be spent on the job and the number of large van cannot exceed the number of small vans. Formulate this problem as a LPP given that the objective is to minimize the cost. Also find the minimum cost. $[Ans: Rs. 2571.43]$
3. A company manufactures two types of screws A and B. All the screws have to pass through a threading machine and a slotting machine. A box of type A screws require 2 min on the threading machine and 3 min on the slotting machine. A box of type B screws requires 8 min on the threading machine and 2 min on the slotting machine. In a week, each machine is available for 60h. On selling these screws, the company gets a profit of Rs.100 per box on type A screws and Rs.170 per box on type B screws. Formulate this problem as LPP given that the objective is to maximize profit. Find the maximum profit. $[Ans: Rs. 138600]$
4. A company manufactures two types of sweaters type A and type B. It cost Rs.360 to make a type A sweater and Rs.120 to make a type B sweater. The company can make at most 300 sweaters and spend at most Rs.72000 a day. The number of sweaters of type B cannot exceed the number of sweaters of type A by more than 100. The company makes a profit of Rs.200 for each sweater of type A and Rs.120 for every sweater of type B. Formulate this problem as a LPP to maximize the profit to the company. Find the maximum profit. $[Ans: Rs. 48000]$
5. A man rides his motorcycle at the speed of 50 km/h. He has to spend Rs.2 per km on petrol. If rides it at a faster speed of 80km/h, the petrol cost increases to Rs.3 per km. He has at most Rs.120 to spend on petrol and one hour’s time. He wishes to find

the maximum distance that he can travel. Express this problem as a linear programming problem. Find the maximum distance covered. $\left[Ans: 54\frac{2}{7}\,km\right]$

6. A company makes 3 model of calculators; A, B , C at the factory I and factory II. The company has orders of at least 6400 calculators of model A, 4000 calculators of model B and 4800 models of calculators C. At factory I, 50 calculators of model A, 50 of model B and 30 of model C are made every day; at factory II, 40 calculators of model A, 20 of model B and 40 of model C are made every day. It cost Rs.12000 and Rs.15000 each day to operate factory I and II, respectively. Find the number of days each factory should operated to minimize the operating costs and still meet the demand. $[Ans: I = 80 days, II = 60\ days]$

PROBABILITY

Questions

1. For a loaded die, the probabilities of outcomes are given as under $P(1) = P(2) = 0.2, P(3) = P(5) = P(6) = 0.1\ and\ P(4) = 0.3$, the die is thrown two times. Let A and B be the events, same number each time and a total score is 10 or more respectively. Determine whether or not A and B are independent. $[Ans: Independent\ events]$
2. The probability that at least one of the two events A and B occurs is 0.6. If A and B occurs simultaneously with probability 0.3, evaluate $P(\bar{A}) + P(\bar{B})$. $[Ans: 1.1]$
3. Explain why the experiment of tossing a coin three times is said to have Binomial distribution.
4. In a dice game, a player pays a stake of Re1 for each throw of a die. She receives Rs.5, if the die shows a 3, Rs.2 if the die shows a 1 or 6 and nothing otherwise, then what is the player's expected profit per throw over a long series of throws? $[Ans: Rs. 0.50]$
5. Three dice are thrown at the same time. Find the probability of getting three two's, if it is known that the sum of the numbers on the dice was six. $\left[\left[Ans: \frac{1}{10}\right]\right]$
6. Suppose 10000 tickets are sold in a lottery each for Re.1. First prize is of Rs.3000 and the second prize is of Rs.2000. There are three third prize of Rs.500 each. If you buy one ticket, then what is your expectation? $[Ans: Rs. 0.65]$
7. A bag contains 4 white and 5 black balls. Another bag contains 9 white and 7 black balls. A ball is transferred from the first bag to the second and then a ball is drawn at a random from the second bag. Find the probability that the ball drawn is white. $\left[Ans: \frac{5}{9}\right]$

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8. A box has 5 blue and 4 red balls. One ball is drawn at random and not replaced. Its colour is also not noted. Then, another ball is drawn at random. What is the probability of second ball being blue. $\left[Ans: \frac{5}{9}\right]$
9. The probability of a man hitting a target is 0.25. If he shoots 7 times, then what is the probability of his hitting at least twice? $\left[Ans: \frac{4547}{8192}\right]$
10. Suppose that 6% of the people with blood group O are left handed and 10% of those with other blood groups are left handed, 30% of the people have blood group O. If a left handed person is selected at a random, what is the probability that he/she will have blood group O? $\left[Ans: \frac{9}{44}\right]$
11. If two natural numbers r and s are drawn one at a time, without replacement from the set $S = \{1, 2, 3, \dots, n\}$, then find $P(r \le p/s \le p)$. Where $p \in S$. $\left[Ans: \frac{p-1}{n-1}\right]$
12. The random variable X can take only the values 0, 1, 2.
    If $P(X = 0) = P(X = 1) = p$ and $E(X^2) = E(X)$, then find the value of p. $\left[Ans: \frac{1}{2}\right]$
13. A and B throw a pair of dice alternately. A wins the game, if he gets a total of 6 and B wins, if she gets a total of 7. If A starts the game, then find the probability of wining the game by A in third throw of the pair of dice. $\left[Ans: \frac{775}{7776}\right]$
14. Two dice are tossed. Find whether the following two events A and B are independent $A = \{(x, y): x + y = 11\}$ and $B = \{(x, y): x \ne 5\}$, where (x, y) denote typical sample point. $[Ans: Not\ independent]$
15. An urn contains m white and n black balls. A ball is drawn at random and is put back into the urn along with k additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. Show that the probability of drawing a white ball now does not depend on k.
16. A shopkeeper sells three types of flower seeds A_1, A_2 and A_3. They are sold as a mixture, where the proportions are 4:4:2, respectively. The germination rates of the three types of seeds are 45%, 60% and 35%. Calculate the probability
    i) Of a randomly chosen seeds to germinate.
    ii) That it will not germinate given that the seed is of type A_3
    iii) That it is of the type of A_2 given that a randomly chosen seed does not germinate. $[Ans: 0.49, 0.65, 0.314]$
17. A letter is known to have come either from TATA NAGAR or from CALCUTTA. On the envelope just two consecutive letters TA are visible. What is the probability that the letter came from TATA NAGAR? $\left[Ans: \frac{7}{11}\right]$
18. There are two bags, one of which contains 3 black and 4 white balls while the other contains 4 black and 3 white balls. A die is thrown. If it shows up 1 or 3, a ball is

taken from the I bag but it shows up any other number a ball is chosen from II bag. Find the probability of choosing a black ball. $\left[Ans: \frac{11}{21}\right]$

19. There are three urns containing 2 white and 3 black balls, 3 white and 2 black balls and 4 white and 1 black balls, respectively. There is an equal probability of each urn being chosen. A ball is drawn at random from the chosen urn and it is found to be white. Find the probability that the ball drawn was from the second urn. $\left[Ans: \frac{1}{3}\right]$
20. By examining the chest X-ray the probability that TB is detected when a person is actually suffering is 0.99. The probability of an healthy person diagnosed to have TB is 0.001. In a certain city, 1 in 1000 people suffers from TB. A person is selected at a random and is diagnosed to have TB. What is the probability that he actually has TB? $\left[Ans: \frac{110}{221}\right]$
21. A bag contains $(2n + 1)$ coins. It is known that n of these coins have a head on both sides whereas the rest of the coins are fair. A coin is picked up at a random from the bag and is tossed. If the probability that the toss result in a head is $\frac{31}{42}$, then determine the value of n. $[Ans: 10]$
22. If A and B are events such that $P(A) = 0.4$, $P(\text{B}) = 0.3$ and $P(A \cup B) = 0.5$ then find $P(B' \cap A)$. $\left[Ans: \frac{1}{5}\right]$
23. If $P(A) = \frac{2}{5}$, $P(B) = \frac{3}{10}$ and $P(A \cap B) = \frac{1}{5}$, then find $P(A'/B').P(B'/A')$. $\left[Ans: \frac{25}{42}\right]$
24. If $P(B) = \frac{3}{5}$, $P(A/B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$, then find $P(A \cup B)' + P(A' \cup B)$. $[Ans: 1]$
25. Two events E and F are independent. If $P(E) = 0.3$ and $P(E \cup F) = 0.5$, then find the value of $P(E/F) - P(F/E)$. $\left[Ans: \frac{1}{70}\right]$
26. Three persons A, B and C fire at a target in turns, starting with A. Their probability of hitting the target are 0.4, 0.3 and 0.2 respectively. Find the probability of two hits. $[Ans: 0.188]$
27. In a college, 30% students fail in physics, 25% fail in mathematics and 10% fail in both. One student is chosen at a random. Find the probability that she fails in physics, if she has failed in mathematics. $\left[Ans: \frac{2}{5}\right]$
28. A and B are two students. Their chances of solving a problem correctly are $\frac{1}{3}$ and $\frac{1}{4}$, respectively. If the probability of their making a common error is $\frac{1}{20}$ and they obtain the same answer, then find the probability of their answer to be correct. $\left[Ans: \frac{10}{13}\right]$

Examples

1. The probability of simultaneous occurrence of at least one of two events A and B is p. If the probability that exactly one of A, B occurs is q , then prove that $P(A') + P(B') = 2 - 2p + q$
2. Find the probability that in 10 throws of a fair die a score which is a multiple of 3 will be obtained in at least 8 of the throws. $\left[Ans: \frac{201}{3^{10}}\right]$
3. A discrete random variable X has the following probability distribution:

X	1	2	3	4	5	6	7
P(X)	C	2C	2C	3C	C^2	$2C^2$	$7C^2+C$

Find the value of C. Also find the mean of the distribution. $\left[Ans: \frac{1}{10}, 3.66\right]$

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