Important Questions(C.B.S.E.)

1. Find the value of $[\hat{\imath}, \hat{\jmath}, \hat{k}]$
2. Find the identity element in the set Q^+ of all positive rational numbers for the operation $*$ defined by $a * b = \frac{3ab}{2}$ for all $a, b \in Q^+$
3. Differentiate: $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$ with respect to x.
4. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 5, |\vec{b}| = 6, |\vec{c}| = 9$, then find angle between $\vec{a}$ and $\vec{b}$.
5. Evaluate $P(A \cup B)$, if $2P(A) = P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$
6. Using properties of determinant, prove that:
$$\begin{vmatrix} 5a & -2a+b & -2a+c \\ -2b+a & 5b & -2b+c \\ -2c+a & -2c+b & 5c \end{vmatrix} = 12(a+b+c)(ab+bc+ca)$$
7. Find the angle of intersection of the curves $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 4$ at the point in the first quadrant.
8. Find the particular solution of the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$ given that $y = 0$ $when$ $x = 1$.
9. Find x such that the four points $A(4,4,4), B(5,x,8), C(5,4,1)$ and $D(7,7,2)$ are coplanar.
10. Using integration find the area of the region: $\{(x,y): 0 \le 2y \le x^2, 0 \le y \le x, 0 \le x \le 3\}$
11. Find the vector equation of the line passing through $(1,2,3)$ and parallel to each of the planes $\vec{r}.(\hat{\imath} - \hat{\jmath} + 2\hat{k}) = 5$ and $\vec{r}.(3\hat{\imath} + \hat{\jmath} + \hat{k}) = 6$. Also find the point of intersection of the line thus obtained with the plane $\vec{r}.(2\hat{\imath} + \hat{\jmath} + \hat{k}) = 4$.
12. A company produces two types of goods, A and B that required gold and silver. Each unit of type A require 3g of silver and 1g of gold while that of B requires 1g of silver and 2g of gold. The company can use at-most 9g of silver and 8g of gold. If each unit of type A brings a profit of $Rs. 40$ and that of type B $Rs. 50$, find the number of units of each type that the company should produce to maximise the profit. Formulate and solve the LPP and find the maximum profit.
13. If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmetric, find the values of $'a'$ and $'b'$.
14. Find the magnitude of each of the two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is $60°$ and their scalar product is $\frac{9}{2}$.

15. Differentiate: $\tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$ with respect to x.
16. Using properties of determinants, prove that:
$$\begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix} = 9(3xyz + xy + yz + zx)$$
17. If $y = \sin(\sin x)$, prove that $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y\cos^2 x = 0$.
18. Find the integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$.
19. A school wants to award its students for regularity and hard work with a total cash award of $Rs. 6{,}000$. If three times the award money for hard work added to that given for regularity amounts to $Rs. 11000$, represent above situations algebraically and find the award money for each value, using matrix method. Suggest two more values which the school must include for award.
20. Find the real solutions of the equation $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1} x, (x > 0)$.
21. Find the coordinates of the foot of perpendicular drawn from a point $A(1, 8, 4)$ to the line joining the points $B(0, -1, 3)$ and $C(2, -3, -1)$.
22. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ are mutually perpendicular vectors of equal magnitudes, find the angles which the vector $2\vec{a} + \vec{b} + 2\,\vec{c}$ makes with the vectors $\vec{a}, \vec{b}$ *and* $\vec{c}$.
23. If $y = (\sec^{-1} x)^2$, then show that $x^2(x^2 - 1)\frac{d^2y}{dx^2} + (2x^3 - x)\frac{dy}{dx} = 2$.
24. Evaluate: $\int_0^{\pi} \frac{x}{1+\sin x} dx$
25. Find the equation of tangent to the curve $x = a\cos\theta + a\theta\sin\theta$, $y = a\sin\theta - a\theta\cos\theta$ at any point θ of the curve. Also show that at any point θ of the curve the normal is at a constant distance from the origin.
26. Given a non-empty set X, consider the binary operation $*$: $P(X) \times P(X) \to P(X)$ given by $A * B = A \cap B \;\forall\, A, B \in P(X)$, where $P(X)$ is the power set of X. Show that $*$ is commutative and associative and X is the identity element for this operation and X is the only invertible element in $P(X)$ with respect to the operation $*$.
27. Find the coordinates of the point of intersection P of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane determined by the points $A(1, -2, 2), B(4, 2, 3)$ *and* $C(3, 0, 2)$
28. A biased die is such that $P(4) = \frac{1}{10}$ and other scores are equally likely. The die is tossed twice. If X is the number of fours obtained, find the variance of X.
29. Find the distance between the line $\vec{r} = 3\hat{i} + 5\hat{j} - 2\hat{k} + \lambda(3\hat{i} + \hat{j} + 3\hat{k})$ and the plane determined by the points $A(1, 1, 0), B(1, 2, 1)$ *and* $C(-2, 2, -1)$.
30. Solve the differential equation $y.e^{\frac{x}{y}}.dx = \left(x.e^{\frac{x}{y}} + y^2\right)dy. (y \neq 0)$
31. Find the co-ordinate of foot of perpendicular drawn from the point $(2, 3, -8)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$

32. Find the value of k for which the function $f(x) = \begin{cases} \frac{\sin x - \cos x}{4x - \pi}, x \neq \frac{\pi}{4} \\ k, x = \frac{\pi}{4} \end{cases}$

33. There are three categories of students in a class of 60 students:
    A: Very hardworking students
    B: Regular but not so hard working
    C: Careless and irregular
    10 students are in category A, 30 in category B and rest in category C. It is found that probability of students of category A, unable to get good marks in the final year examination is 0.002, of category B it is 0.02 and of category C, this probability is 0.20. A student selected at random was found to be the one who could not get good marks in examination. Find the probability that this student is of category C. What values need to be developed in students of category C?

34. Using integration, find the area of the following region: $\{(x, y): y^2 \geq ax, x^2 + y^2 \leq 2ax, a > 0\}$.

35. Let $A = \{1, 2, 3, 4\}$. Let R be the equivalence relation on $A \times A$ defined by $(a, b)R(c, d)$ iff $a + d = b + c$. Find the equivalence class $[(1, 3)]$.

36. If $A = [a_{ij}]$ is a matrix of order 2×2, such that $|A| = -15$ and C_{ij} represents the cofactor of a_{ij}, then find $a_{21}C_{21} + a_{22}C_{22}$.

37. If $4\sin^{-1} x + \cos^{-1} x = \pi$, then find the value of x.

38. Find the inverse of the matrix $\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$. Hence, find the matrix P satisfying the matrix equation $P\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

39. Prove that if $\frac{1}{2} \leq x \leq 1$, then $\cos^{-1} x + \cos^{-1}\left[\frac{x}{2} + \frac{\sqrt{3 - 3x^2}}{2}\right] = \frac{\pi}{3}$

40. Find the approximate change in the value of $\frac{1}{x^2}$, when $x = 2\ to\ x = 2.002$.

41. Find : $\int e^x \frac{\sqrt{1 + \sin 2x}}{1 + \cos 2x} dx$

42. Verify that $ax^2 + by^2 = 1$ is a solution of the differential equation $x(yy_2 + y_1^2) = yy_1$

43. If $\Delta = \begin{vmatrix} 1 & a & a^2 \\ a & a^2 & 1 \\ a^2 & 1 & a \end{vmatrix} = -4$, then find the value of $\begin{vmatrix} a^3 - 1 & 0 & a - a^4 \\ 0 & a - a^4 & a^3 - 1 \\ a - a^4 & a^3 - 1 & 0 \end{vmatrix}$

44. Find $'a'\ and\ 'b'$ if the function given by $f(x) = \begin{cases} ax^2 + b, if\ x < 1 \\ 2x + 1, if\ x \geq 1 \end{cases}$ is differentiable at $x = 1$.

45. Determine the values of $'a'\ and\ 'b'$ such that the following function is continuous at $x = 0$: $f(x) = \begin{cases} \frac{x + \sin x}{\sin(a+1)x}, if\ -\pi < x < 0 \\ 2, if\ x = 0 \\ 2\frac{e^{\sin bx} - 1}{bx}, if\ x > 0 \end{cases}$

46. If $y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$, then prove that $x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$
47. Find the equations of the tangents to the curve $y = (x^2 - 1)(x - 2)$ at the points where the curve intersects the $x - axis$.
48. Find the intervals in which the function $f(x) = -3\log(1+x) + 4\log(2+x) - \frac{4}{2+x}$ is strictly increasing or strictly decreasing.
49. A person wants to plant some trees in his community park. The local nursery has to perform this task. It charges the cost of planting trees by the following formula $C(x) = x^3 - 45x^2 + 600x$, where x is the number of trees and $C(x)$ is the cost of planting x trees in rupees. The local authority has imposed a restriction that it can plant 10 to 20 trees in one community park for a fair distribution. For how many trees should the person places the order so that he has to spend the least amount? How much is the least amount? Use calculus to answer these questions. Which value is being exhibited by the person?
50. Find $\int \frac{\sec x}{1 + co\sec x} dx$
51. Find the particular solution of the differential equation: $ye^x dx = (y^3 + 2xe^y)dy, y(0) = 1$.
52. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$ and hence show that $[\vec{a},\ \vec{b},\ \vec{c}] = 0$.
53. Find the equation of the line which intersects the lines $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and passes through the point $(1, 1, 1)$.
54. Bag I contains 1 white, 2 black and 3 red balls; Bag II contains 2 white, 1 black and 1 red balls; Bag III contains 4 white, 3 black and 2 red balls. A bag is chosen at random and two balls are drawn from it with replacement. They happen to be one white and one red. What is the probability that they came from bag III.
55. If the function $f: R \to R$ be defined by $f(x) = 2x - 3$ and $g: R \to R$ by $g(x) = x^3 + 5$. Then find fog and show that fog is invertible. Also find $(fog)^{-1}$, hence find $(fog)^{-1}(9)$.
56. A binary operation $*$ is defined on the set R of real numbers by $a * b = \begin{cases} a, if b = 0 \\ |a| + b, if\ b \neq 0 \end{cases}$. If at least one of $a\ and\ b$ is 0, then prove that $a * b = b * a$. Check whether $*$ is commutative. Find the identity element for $*$ if it is exists.
57. If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations: $3x + 4y + 7z = 14, 2x - y + 3z = 4, x + 2y - 3z = 0$.

58. If $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$, find the inverse of A using elementary row transformation and hence solve the following matrix equation $XA = [1 \quad 0 \quad 1]$

59. Using integration, find the area in the first quadrant bounded by the curve $y = x|x|$, the circle $x^2 + y^2 = 2$ and $y - axis$.

60. Evaluate the following: $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x+\frac{\pi}{4}}{2-\cos 2x} dx$

61.

A company produces two different products. One of them needs 1/4 of an hour of assembly work per unit, 1/8 of an hour in quality control work and Rs1.2 in raw materials. The other product requires 1/3 of an hour of assembly work per unit, 1/3 of an hour in quality control work and Rs 0.9 in raw materials. Given the current availability of staff in the company, each day there is at most a total of 90 hours available for assembly and 80 hours for quality control. The first product described has a market value (sale price) of Rs 9 per unit and the second product described has a market value (sale price) of Rs 8 per unit. In addition, the maximum amount of daily sales for the first product is estimated to be 200 units, without there being a maximum limit of daily sales for the second product. Formulate and solve graphically the LPP and find the maximum profit.

62.

A and B are square matrices of order 3 each, |A| = 2 and |B| = 3. Find |3AB|

63.

What is the distance of the point (p, q, r) from the x-axis?

64.

Let f : R → R be defined by f(x) = $3x^2 - 5$ and g : R → R be defined by g(x) = $\frac{x}{x^2+1}$. Find *gof*

65.

How many equivalence relations on the set {1,2,3} containing (1,2) and (2,1) are there in all ? Justify your answer.

66.

Let l_i, m_i, n_i ; i = 1, 2, 3 be the direction cosines of three mutually perpendicular vectors in space. Show that AA' = I_3, where A = $\begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$.

67.

Find the sum of the order and the degree of the following differential equations:

$$\frac{d^2y}{dx^2} + \sqrt[3]{\frac{dy}{dx}} + (1 + x) = 0$$

68.

If A + B + C = π, then find the value of

$$\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix}$$

69.

It is given that for the function $f(x) = x^3 - 6x^2 + ax + b$ Rolle's theorem holds in [1, 3] with $c = 2 + \frac{1}{\sqrt{3}}$. Find the values of 'a' and 'b'

70.

A bag contains (2n +1) coins. It is known that 'n' of these coins have a head on both its sides whereas the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$, find the value of 'n'.

71.

Find: $\int \frac{\sin x}{\sin^3 x + \cos^3 x} dx$

72.

Determine whether the operation $*$ define below on $\mathbb{Q}$ is binary operation or not.

$a * b = ab+1$

If yes, check the commutative and the associative properties. Also check the existence of identity element and the inverse of all elements in $\mathbb{Q}$.

73.

Find the value of x, y and z, if A = $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies $A' = A^{-1}$

74.

Find $\frac{dy}{dx}$, if $y = e^{\sin^2 x}\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}$

75.

Give an example of a skew symmetric matrix of order 3.

76.

Using derivative, find the approximate percentage increase in the area of a circle if its radius is increased by 2% .

77.

Find the derivative of $f(e^{\tan x})$ w.r.t. to x at x = 0. It is given that $f'(1) = 5$.

78.

Without expanding the determinant at any stage, prove that $\begin{vmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{vmatrix} = 0$.

79.

If the following function is differentiable at x = 2, then find the values of a and b.

$$f(x) = \begin{cases} x^2, \text{ if } x \le 2 \\ ax + b, \text{ if } x > 2 \end{cases}.$$

80.

Evaluate the following indefinite integral: $\int \frac{1}{\sin x - \sin 2x} dx$.

81.

Evaluate the following definite integral: $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$.

82.

A problem in mathematics is given to 4 students A, B, C, D. Their chances of solving the problem, respectively, are 1/3, 1/4, 1/5 and 2/3. What is the probability that (i) the problem will be solved? (ii) at most one of them will solve the problem?

83.

Using integration, find the area bounded by the tangent to the curve $4y = x^2$ at the point (2, 1) and the lines whose equations are $x = 2y$ and $x = 3y - 3$.

84.

Find the distance of the point $3\hat{i} - 2\hat{j} + \hat{k}$ from the plane $3x + y - z + 2 = 0$ measured parallel to the line $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-1}{1}$. Also, find the foot of the perpendicular from the given point upon the given plane.

85.

A Bag I contains 5 red and 4 white balls and a Bag II contains 3 red and 3 white balls. Two balls are transferred from the Bag I to the Bag II and then one ball is drawn from the Bag II. If the ball drawn from the Bag II is red, then find the probability that one red ball and one white ball are transferred from the Bag I to the Bag II.

86.

Evaluate :

$$\int \frac{\sin x - x\cos x}{x(x + \sin x)}\,dx$$

87.

Evaluate :

$$\int \frac{x^3}{(x-1)(x^2+1)}\,dx$$

88.

Evaluate :

$$\int_0^{\pi/2} \frac{\cos^2 x\,dx}{1 + 3\sin^2 x}$$

89.

Evaluate :

$$\int_0^{\pi/4} \left(\frac{\sin x + \cos x}{3 + \sin 2x}\right) dx$$

✆ +91- 9971750366, 8447026670 ✉ingeniusinfinity@gmail.com 🌐: https://ingeniousinfinity.com

90.

A man takes a step forward with probability 0·4 and backward with probability 0·6. Find the probability that at the end of 5 steps, he is one step away from the starting point.

91.

Tangent to the circle $x^2 + y^2 = 4$ at any point on it in the first quadrant makes intercepts OA and OB on x and y axes respectively, O being the centre of the circle. Find the minimum value of (OA + OB).

92.

If the area bounded by the parabola $y^2 = 16ax$ and the line $y = 4mx$ is $\frac{a^2}{12}$ sq. units, then using integration, find the value of m.

93.

Find the equation of a plane passing through the point P(6, 5, 9) and parallel to the plane determined by the points A(3, –1, 2), B(5, 2, 4) and C(–1, –1, 6). Also find the distance of this plane from the point A.

94.

Find a vector of magnitude $\sqrt{171}$ which is perpendicular to both of the vectors $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$.

95.

Find the angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$.

96.

If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then for any natural number n, find the value of Det (A^n).

97.

Discuss the continuity and differentiability of the function $f(x) = |x| + |x-1|$ in the interval (–1, 2).

98.

There are 2 families A and B. There are 4 men, 6 women and 2 children in family A, and 2 men, 2 women and 4 children in family B. The recommended daily amount of calories is 2400 for men, 1900 for women, 1800 for children and 45 grams of proteins for men, 55 grams for women and 33 grams for children. Represent the above information using matrices. Using matrix multiplication, calculate the total requirement of calories and proteins for each of the 2 families. What awareness can you create among people about the balanced diet from this question ?

99.

Evaluate :

$$\tan\left\{2\tan^{-1}\left(\frac{1}{5}\right)+\frac{\pi}{4}\right\}$$

100.

Write the sum of the order and degree of the following differential equation :

$$\frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^3\right\}=0$$

101.

Write the integrating factor of the following differential equation :

$$(1+y^2)+(2xy-\cot y)\frac{dy}{dx}=0$$

102.

If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors, then find the value of $|2\hat{a}+\hat{b}+\hat{c}|$.

103.

The equations of a line are $5x-3=15y+7=3-10z$. Write the direction cosines of the line.

104.

To promote the making of toilets for women, an organisation tried to generate awareness through (i) house calls (ii) letters, and (iii) announcements. The cost for each mode per attempt is given below :

(i) ₹ 50

(ii) ₹ 20

(iii) ₹ 40

The number of attempts made in three villages X, Y, and Z are given below :

	(i)	(ii)	(iii)
X	400	300	100
Y	300	250	75
Z	500	400	150

Find the total cost incurred by the organisation for the three villages separately, using matrices.

Write one value generated by the organisation in the society.

105.

If $f(x) = \sqrt{x^2 + 1}$; $g(x) = \dfrac{x+1}{x^2+1}$ and $h(x) = 2x - 3$, then find $f'[h'\{g'(x)\}]$.

106.

Find :

$$\int_0^{\pi/4} \frac{dx}{\cos^3 x \sqrt{2 \sin 2x}}$$

107.

Find :

$$\int \frac{\log x}{(x+1)^2}\, dx$$

108.

For 6 trials of an experiment, let X be a binomial variate which satisfies the relation $9P(X = 4) = P(X = 2)$. Find the probability of success.

109.

Three cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spades. Hence find the mean of the distribution.

110.

Consider $f : \mathbb{R}_+ \rightarrow [-9, \infty]$ given by $f(x) = 5x^2 + 6x - 9$. Prove that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{54 + 5y} - 3}{5}\right)$.

111.

A binary operation * is defined on the set x = R – { – 1 } by

$$x * y = x + y + xy, \ \forall x, y \in X.$$

Check whether * is commutative and associative. Find its identity element and also find the inverse of each element of X.

112.

Find the value of p for when the curves $x^2 = 9p(9 - y)$ and $x^2 = p(y + 1)$ cut each other at right angles.

113.

A company manufactures three kinds of calculators : A, B and C in its two factories I and II. The company has got an order for manufacturing at least 6400 calculators of kind A, 4000 of kind B and 4800 of kind C. The daily output of factory I is of 50 calculators of kind A, 50 calculators of kind B, and 30 calculators of kind C. The daily output of factory II is of 40 calculators of kind A, 20 of kind B and 40 of kind C. The cost per day to run factory I is ₹ 12,000 and of factory II is ₹ 15,000. How many days do the two factories have to be in operation to produce the order with the minimum cost ? Formulate this problem as an LPP and solve it graphically.

114.

Find the sum of the degree and the order for the following differential equation :

$$\frac{d}{dx}\left[\left(\frac{d^2y}{dx^2}\right)^4\right] = 0$$

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115.

The vectors $\vec{a} = 3\hat{i} + x\hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} + y\hat{k}$ are mutually perpendicular. If $|\vec{a}| = |\vec{b}|$, then find the value of y.

116.

If $|\vec{a}| = a$, then find the value of the following :

$$|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$$

117.

A trust caring for handicapped children gets ₹ 30,000 every month from its donors. The trust spends half of the funds received for medical and educational care of the children and for that it charges 2% of the spent amount from them, and deposits the balance amount in a private bank to get the money multiplied so that in future the trust goes on functioning regularly. What percent of interest should the trust get from the bank to get a total of ₹ 1,800 every month ?

Use matrix method, to find the rate of interest. Do you think people should donate to such trusts ?

118.

Let $f(x) = x - |x - x^2|$, $x \in [-1, 1]$. Find the point of discontinuity, (if any), of this function on $[-1, 1]$.

119.

$$\text{If } y = \log\left(\frac{x}{a + bx}\right)^x, \text{ prove that } x^3 \frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2.$$

120.

Evaluate :

$$\int_0^{\pi/2} \left(\frac{5\sin x + 3\cos x}{\sin x + \cos x}\right) dx$$

121.

Find :

$$\int \left[\log\log x + \frac{1}{(\log x)^2}\right] dx$$

122.

Find :

$$\int \frac{x\,dx}{1 + x \tan x}$$

123.

Find :

$$\int \frac{x^4}{(x-1)(x^2+1)}\,dx$$

124.

Find a unit vector perpendicular to the plane of triangle ABC, where the coordinates of its vertices are A(3, – 1, 2), B(1, – 1, – 3) and C(4, – 3, 1).

125.

Find the shortest distance between the lines $x + 1 = 2y = -12z$ and $x = y + 2 = 6z - 6$.

126.

From the point P(a, b, c), perpendiculars PL and PM are drawn to YZ and ZX planes respectively. Find the equation of the plane OLM.

127.

In 3 trials of a binomial distribution, the probability of exactly 2 successes is 9 times the probability of 3 successes. Find the probability of success in each trial.

128.

On the set {0, 1, 2, 3, 4, 5, 6}, a binary operation $*$ is defined as :

$$a * b = \begin{cases} a + b, & \text{if } a + b < 7 \\ a + b - 7, & \text{if } a + b \geq 7 \end{cases}$$

Write the operation table of the operation $*$ and prove that zero is the identity for this operation and each element $a \neq 0$ of the set is invertible with $7 - a$ being the inverse of a.

129.

Find the area of the region $\{(x, y) : x^2 + y^2 \leq 4,\ x + y \geq 2\}$, using the method of integration.

✆ +91- 9971750366, 8447026670 ✉ingeniusinfinity@gmail.com 🌐: https://ingeniousinfinity.com

130.

Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ with its vertex at one end of the major axis.

131.

$(x^2 + y^2)\, dy = xy\, dx$. If $y(1) = 1$ and $y(x_0) = e$, then find the value of x_0.

132.

Find the value of $\vec{a} \cdot \vec{b}$, if $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $|\vec{a} \times \vec{b}| = 16$.

133.

Prove the following :

$$\sin\left[\tan^{-1}\left(\frac{1-x^2}{2x}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right] = 1, \quad 0 < x < 1.$$

134.

Examine the following function f(x) for continuity at $x = 1$ and differentiability at $x = 2$.

$$f(x) = \begin{cases} 5x - 4 & , \quad 0 < x < 1 \\ 4x^2 - 3x & , \quad 1 < x < 2 \\ 3x + 4 & , \quad x \geq 2 \end{cases}$$

135.

If $\frac{x}{x-y} = \log \frac{a}{x-y}$, then prove that $\frac{dy}{dx} = 2 - \frac{x}{y}$.

136.

Find :

$$\int \frac{dx}{x^3(x^5+1)^{3/5}}$$

137.

Three schools A, B and C want to award their selected students for the values of Honesty, Regularity and Hard work. Each school decided to award a sum of ₹ 2,500, ₹ 3,100, ₹ 5,100 per student for the respective values. The number of students to be awarded by the three schools is given below in the table :

Values \ School	A	B	C
Honesty	3	4	6
Regularity	4	5	2
Hard work	6	3	4

Find the total money given in awards by the three schools separately, using matrices.
Apart from the above given values, suggest one more value which should be considered for giving award.

138.

Let A = { –1, 0, 1, 2}, B = { – 4, – 2, 0, 2} and f, g : A → B be functions defined by $f(x) = x^2 - x,\ x \in A$ and $g(x) = 2\left|x - \frac{1}{2}\right| - 1,\ x \in A$. Find gof (x) and hence show that f = g = gof.

139.

Find the absolute maximum and absolute minimum values of the function f given by $f(x) = \cos^2 x + \sin x,\ x \in [0, \pi]$.

140.

Prove that :

$$2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right) = \cos^{-1}\left(\frac{a\cos x + b}{a + b\cos x}\right)$$

141.

Using the properties of determinants, prove the following :

$$\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(1-x) & x(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix} = 6x^2(1-x^2)$$

142.

If $x = \alpha \sin 2t\,(1 + \cos 2t)$ and $y = \beta \cos 2t\,(1 - \cos 2t)$, show that $\frac{dy}{dx} = \frac{\beta}{\alpha} \tan t$.

143.

Find the derivative of the following function f(x) w.r.t. x, at x = 1 :

$$\cos^{-1}\left[\sin \sqrt{\frac{1+x}{2}}\,\right] + x^x$$

144.

Evaluate :

$$\int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}}\, dx$$

145.

Evaluate :

$$\int_0^{3/2} |x \cdot \cos(\pi x)|\, dx$$

146.

Find :

$$\int \frac{x^3 - 1}{x^3 + x}\, dx$$

147.

Find the acute angle between the plane $5x - 4y + 7z - 13 = 0$ and the y-axis.

148.

If the function $f(x) = 2x^3 - 9mx^2 + 12m^2x + 1$, where $m > 0$ attains its maximum and minimum at p and q respectively such that $p^2 = q$, then find the value of m.

149.

Find the differential equation for all the straight lines, which are at a unit distance from the origin.

150.

Find the direction ratios of the normal to the plane, which passes through the points (1, 0, 0) and (0, 1, 0) and makes angle $\frac{\pi}{4}$ with the plane $x + y = 3$. Also find the equation of the plane.

151.

The postmaster of a local post office wishes to hire extra helpers during the Deepawali season, because of a large increase in the volume of mail handling and delivery. Because of the limited office space and the budgetary conditions, the number of temporary helpers must not exceed 10. According to past experience, a man can handle 300 letters and 80 packages per day, on the average, and a woman can handle 400 letters and 50 packets per day. The postmaster believes that the daily volume of extra mail and packages will be no less than 3400 and 680 respectively. A man receives ₹ 225 a day and a woman receives ₹ 200 a day. How many men and women helpers should be hired to keep the pay-roll at a minimum ? Formulate an LPP and solve it graphically.

152.

Write a differential equation for $y = A \cos \alpha x + B \sin \alpha x$, where A and B are arbitrary constants.

153.

Write the direction cosines of the normal to the plane $3x + 4y + 12z = 52$.

154.

There are 3 families A, B and C. The number of men, women and children in these families are as under :

	Men	Women	Children
Family A	2	3	1
Family B	2	1	3
Family C	4	2	6

Daily expenses of men, women and children are ₹ 200, ₹ 150 and ₹ 200 respectively. Only men and women earn and children do not. Using matrix multiplication, calculate the daily expenses of each family. What impact does more children in the family create on the society ?

155.

If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$, x, y, z, > 0, then find the value of xy + yz + zx.

156.

If $X\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$, then find the matrix X.

157.

If function $f(x) = |x-3| + |x-4|$, then show that f(x) is not differentiable at x = 3 and x = 4.

158.

If $y = x^{e^{-x^2}}$, find $\frac{dy}{dx}$.

159.

If $y = \sqrt{x+1} - \sqrt{x-1}$, prove that $(x^2-1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - \frac{1}{4}y = 0$.

160.

Find :

$$\int \frac{1-\cos x}{\cos x\,(1+\cos x)}\,dx$$

161.

Evaluate :

$$\int x \cdot \sin^{-1} x \, dx$$

162.

Find $\int_0^2 (x^2 + e^{2x+1})\, dx$ as the limit of a sum.

163.

Evaluate :

$$\int_0^{\pi} \frac{x \tan x}{\sec x \cdot \operatorname{cosec} x}\, dx$$

164.

Show that the lines $\frac{x-1}{3} = \frac{y-1}{-1}, z+1=0$ and $\frac{x-4}{2} = \frac{z+1}{3}, y=0$ intersect each other. Also find their point of intersection.

165.

Let P(3, 2, 6) be a point in the space and Q be a point on the line $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$, then find the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane $x - 4y + 3z = 1$.

166.

Find the vector and cartesian equations of the plane which bisects the line joining the points (3, – 2, 1) and (1, 4, – 3) at right angles.

167.

From a set of 100 cards numbered 1 to 100, one card is drawn at random. Find the probability that the number on the card is divisible by 6 or 8, but not by 24.

168.

Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is divisible by 2 } is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other, but no element of {1, 3, 5} is related to any element of {2, 4}.

169.

Using integration, find the area bounded by the curves $y = |x - 1|$ and $y = 3 - |x|$.

170.

Find the point on the curve $y = \dfrac{x}{1 + x^2}$, where the tangent to the curve has the greatest slope.

171.

Solve the following differential equation, given that y = 0, when $x = \dfrac{\pi}{4}$:

$$\sin 2x \frac{dy}{dx} - y = \tan x$$

172.

A dealer deals in two items only – item A and item B. He has ₹ 50,000 to invest and a space to store at most 60 items. An item A costs ₹ 2,500 and an item B costs ₹ 500. A net profit to him on item A is ₹ 500 and on item B ₹ 150. If he can sell all the items that he purchases, how should he invest his amount to have maximum profit ? Formulate an LPP and solve it graphically.

173.

If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, using properties of determinants find the value of $f(2x) - f(x)$.

174.

Evaluate : $\displaystyle\int_{-\pi}^{\pi} (\cos ax - \sin bx)^2 \, dx$

175.

A bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag B. If two balls are drawn at random (without replacement) from the selected bag, find the probability of one of them being red and another black.

176.

If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, find $(\vec{r} \times \hat{i}) \cdot (\vec{r} \times j) + xy$

177.

If $\sin[\cot^{-1}(x+1)] = \cos(\tan^{-1}x)$, then find x.

178.

If $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$, then find x.

179.

If $x = a\cos\theta + b\sin\theta$, $y = a\sin\theta - b\cos\theta$, show that $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$.

180.

Let N denote the set of all natural numbers and R be the relation on N × N defined by (a, b) R (c, d) if $ad(b+c) = bc(a+d)$. Show that R is an equivalence relation.

181.

Using integration find the area of the triangle formed by positive x-axis and tangent and normal to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$.

182.

Evaluate $\int_1^3 (e^{2-3x} + x^2 + 1)dx$ as a limit of a sum.

183.

Solve the differential equation :

$$(\tan^{-1}y - x)dy = (1 + y^2)dx.$$

184.

If lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then find the value of k and hence find the equation of the plane containing these lines.

185.

If A and B are two independent events such that $P(\bar{A} \cap B) = \frac{2}{15}$ and $P(A \cap \bar{B}) = \frac{1}{6}$, then find P(A) and P(B).

186.

Find the local maxima and local minima, of the function $f(x) = \sin x - \cos x$, $0 < x < 2\pi$. Also find the local maximum and local minimum values.

187.

Find the differential equation representing the family of curves $v = \frac{A}{r} + B$, where A and B are arbitrary constants.

188.

Write the solution of the differential equation

$$\frac{dy}{dx} = 2^{-y}.$$

189.

Evaluate :

$$\int \frac{x^2}{(x^2+4)(x^2+9)}\,dx$$

190.

Evaluate :

$$\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x}\,dx$$

191.

Three machines E_1, E_2 and E_3 in a certain factory producing electric bulbs, produce 50%, 25% and 25% respectively, of the total daily output of electric bulbs. It is known that 4% of the bulbs produced by each of machines E_1 and E_2 are defective and that 5% of those produced by machine E_3 are defective. If one bulb is picked up at random from a day's production, calculate the probability that it is defective.

192.

The two vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represent the two side vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively of triangle ABC. Find the length of the median through A.

193.

Find the equation of a plane which passes through the point (3, 2, 0) and contains the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$.

194.

If $\tan^{-1}\left(\frac{1}{1+1.2}\right)+\tan^{-1}\left(\frac{1}{1+2.3}\right)+\ldots+\tan^{-1}\left(\frac{1}{1+n.(n+1)}\right)=\tan^{-1}\theta$, then find the value of θ.

195.

Find whether the following function is differentiable at x = 1 and x = 2 or not :

$$f(x)=\begin{cases} x, & x<1 \\ 2-x, & 1\le x\le 2 \\ -2+3x-x^2, & x>2 \end{cases}$$

196.

Evaluate :

$$\int e^{2x}.\sin(3x+1)\,dx$$

197.

Let $f : N \to \mathbb{R}$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : N \to S$, where S is the range of f, is invertible. Also find the inverse of f.

198.

Using integration, find the area of the region bounded by the line $x - y + 2 = 0$, the curve $x = \sqrt{y}$ and y-axis.

199.

Find the minimum value of (ax + by), where $xy = c^2$.

200.

Find the coordinates of a point of the parabola $y = x^2 + 7x + 2$ which is closest to the straight line $y = 3x - 3$.

201.

Find the distance of the point (1, – 2, 3) from the plane $x - y + z = 5$ measured parallel to the line whose direction cosines are proportional to 2, 3, – 6.

202.

Write the differential equation obtained by eliminating the arbitrary constant C in the equation representing the family of curves $xy = C\cos x$.

203.

Two schools A and B decided to award prizes to their students for three values, team spirit, truthfulness and tolerance at the rate of ₹ x, ₹ y and ₹ z per student respectively. School A, decided to award a total of ₹ 1,100 for the three values to 3, 1 and 2 students respectively while school B decided to award ₹ 1,400 for the three values to 1, 2 and 3 students respectively. If one prize for all the three values together amount to ₹ 600 then

(i) Represent the above situation by a matrix equation after forming linear equations.

(ii) Is it possible to solve the system of equations so obtained using matrices ?

(iii) Which value you prefer to be rewarded most and why ?

204.

Using properties of determinants, prove that

$$\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix} = -2$$

205.

Write the vector equation of the line passing through (1, 2, 3) and perpendicular to the plane $\vec{r} \cdot (\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$.

206.

In the interval $\pi/2 < x < \pi$, find the value of x for which the matrix $\begin{pmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{pmatrix}$ is singular.

207.

Write the integrating factor of the differential equation

$$\sqrt{x}\frac{dy}{dx} + y = e^{-2\sqrt{x}}.$$

208.

Find $\int_{0}^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx.$

209.

Prove that $\left[\vec{a}, \vec{b}+\vec{c}, \vec{d}\right] = \left[\vec{a}, \vec{b}, \vec{d}\right] + \left[\vec{a}, \vec{c}, \vec{d}\right]$

210.

For what value of λ the function defined by $f(x) = \begin{cases} \lambda(x^2+2), & \text{if } x \leq 0 \\ 4x+6, & \text{if } x > 0 \end{cases}$ is continuous at $x = 0$? Hence check the differentiability of $f(x)$ at $x = 0$.

211.

If $x = ae^t (\sin t + \cos t)$ and $y = ae^t (\sin t - \cos t)$, prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

212.

Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is divisible by 2} is an equivalence relation. Write all the equivalence classes of R.

213.

In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$, what is the probability that the student knows the answer given that he answered it correctly ?

214.

If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}+\vec{b}| = |\vec{a}|$, then prove that vector $2\vec{a}+\vec{b}$ is perpendicular to vector $\vec{b}$.

215.

Two institutions decided to award their employees for the three values of resourcefulness, competence and determination in the form of prizes at the rate of ₹ x, ₹ y and ₹ z respectively per person. The first institution decided to award respectively 4, 3 and 2 employees with a total prize money of ₹ 37,000 and the second institution decided to award respectively 5, 3 and 4 employees with a total prize money of ₹ 47,000. If all the three prizes per person together amount to ₹ 12,000, then using matrix method find the value of x, y and z.

What values are described in this question ?

+91- 9971750366, 8447026670 ingeniusinfinity@gmail.com : https://ingeniousinfinity.com

216.

In a group of 400 people, 160 are smokers and non-vegetarian, 100 are smokers and vegetarian and the remaining are non-smokers and vegetarian. The probabilities of getting a special chest disease are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the disease. What is the probability that the selected person is a smoker and non-vegetarian ? What value is reflected in this question ?

217.

For a matrix A of order n, given that $|A| = 4$. If $|\text{adj } A| = 16$, find the value of n.

218.

Find the co-ordinates of the image of the point P(1, 3, 4) in the plane $2x - y + z + 3 = 0$. Also find the length of PP'.

219.

Write the number of all possible matrices of order 2×2 with each entry 1, 2 or 3.

220.

Find the coordinates of the point where the line through the points A(3, 4, 1) and B(5, 1, 6) crosses the XZ plane. Also find the angle which this line makes with the XZ plane.

221.

The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$.

Find the two unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram.

222.

In a game, a man wins ₹ 5 for getting a number greater than 4 and loses ₹ 1 otherwise, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a number greater than 4. Find the expected value of the amount he wins/loses.

223.

A bag contains 4 balls. Two balls are drawn at random (without replacement) and are found to be white. What is the probability that all balls in the bag are white ?

224.

The equation of tangent at (2, 3) on the curve $y^2 = ax^3 + b$ is $y = 4x - 5$. Find the values of a and b.

225.

Solve the equation for x : $\sin^{-1}x+\sin^{-1}(1-x)=\cos^{-1}x$

226.

A trust invested some money in two type of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received ₹ 2,800 as interest. However, if trust had interchanged money in bonds, they would have got ₹ 100 less as interest. Using matrix method, find the amount invested by the trust. Interest received on this amount will be given to Helpage India as donation. Which value is reflected in this question ?

227.

Five bad oranges are accidently mixed with 20 good ones. If four oranges are drawn one by one successively with replacement, then find the probability distribution of number of bad oranges drawn. Hence find the mean and variance of the distribution.

228.

Show that the binary operation * on $A=\mathbf{R}-\{-1\}$ defined as $a*b=a+b+ab$ for all a, b ϵ A is commutative and associative on A. Also find the identity element of * in A and prove that every element of A is invertible.

229.

Using properties of determinants, show that ΔABC is isosceles if :

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\cos A & 1+\cos B & 1+\cos C \\ \cos^2 A+\cos A & \cos^2 B+\cos B & \cos^2 C+\cos C \end{vmatrix}=0$$

230.

A shopkeeper has 3 varieties of pens 'A', 'B' and 'C'. Meenu purchased 1 pen of each variety for a total of ₹ 21. Jeevan purchased 4 pens of 'A' variety, 3 pens of 'B' variety and 2 pens of 'C' variety for ₹ 60. While Shikha purchased 6 pens of 'A' variety, 2 pens of 'B' variety and 3 pens of 'C' variety for ₹ 70. Using matrix method, find cost of each variety of pen.

231.

Write the coordinates of the point which is the reflection of the point (α, β, γ) in the XZ-plane.

232.

If $|\vec{a}|=4, |\vec{b}|=3$ and $\vec{a}.\vec{b}=6\sqrt{3}$, then find the value of $|\vec{a}\times\vec{b}|$.

233.

On her birthday Seema decided to donate some money to children of an orphanage home. If there were 8 children less, every one would have got ₹ 10 more. However, if there were 16 children more, every one would have got ₹ 10 less. Using matrix method, find the number of children and the amount distributed by Seema. What values are reflected by Seema's decision ?

234.

Find the equation of the tangent line to the curve $y=\sqrt{5x-3}-5$, which is parallel to the line $4x-2y+5=0$.

235.

Solve the differential equation :

$$x\frac{dy}{dx}+y-x+xy\cot x=0;\ x\neq 0.$$

236.

Show that the lines $\frac{x-1}{3}=\frac{y-1}{-1}=\frac{z+1}{0}$ and $\frac{x-4}{2}=\frac{y}{0}=\frac{z+1}{3}$ intersect.

Find their point of intersection.

237.

A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6
P(X)	C	2C	2C	3C	C^2	$2C^2$	$7C^2+C$

Find the value of C and also calculate mean of the distribution.

238.

Show that the relation R defind by (a, b) R (c, d) $\Rightarrow a+d=b+c$ on the $A\times A$, where $A=\{1, 2, 3, \ldots\ldots\ldots, 10\}$ is an equivalence relation. Hence write the equivalence class [(3, 4)]; a, b, c, d $\in$ A.

239.

Using elementary row operations find the inverse of matrix $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$

and hence solve the following system of equations $3x - 3y + 4z = 21$, $2x - 3y + 4z = 20$, $-y + z = 5$.

240.

Using integration, find the area of the triangle formed by negative x-axis and tangent and normal to the circle $x^2 + y^2 = 9$ at $\left(-1, 2\sqrt{2}\right)$.

241.

A, B and C throw a pair of dice in that order alternately till one of them gets a total of 9 and wins the game. Find their respective probabilities of winning, if A starts first.

242.

A company manufactures two types of cardigans : type A and type B. It costs ₹ 360 to make a type A cardigan and ₹ 120 to make a type B cardigan. The company can make at most 300 cardigans and spend at most ₹ 72,000 a day. The number of cardigans of type B cannot exceed the number of cardigans of type A by more than 200. The company makes a profit of ₹ 100 for each cardigan of type A and ₹ 50 for every cardigan of type B.

Formulate this problem as a linear programming problem to maximise the profit to the company. Solve it graphically and find maximum profit.

243.

If vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = \frac{1}{2}$, $|\vec{b}| = \frac{4}{\sqrt{3}}$ and $|\vec{a} \times \vec{b}| = \frac{1}{\sqrt{3}}$, then find $|\vec{a} \cdot \vec{b}|$.

244.

If $\vec{a}$ and $\vec{b}$ are unit vectors, then what is the angle between $\vec{a}$ and $\vec{b}$ for $\vec{a} - \sqrt{2}\,\vec{b}$ to be a unit vector ?

245.

If A is a square matrix such that $|A| = 5$, write the value of $|AA^T|$.

246.

If $A = \begin{pmatrix} 0 & 3 \\ 2 & -5 \end{pmatrix}$ and $KA = \begin{pmatrix} 0 & 4a \\ -8 & 5b \end{pmatrix}$ find the values of k and a.

247.

Find equation of normal to the curve $ay^2 = x^3$ at the point whose x coordinate is am^2.

248.

Find : $\int \frac{1 - \sin x}{\sin x (1 + \sin x)} dx$

249.

Evaluate : $\int_0^1 \cot^{-1}\left(1 - x + x^2\right) dx$

250.

Solve the differential equation : $(x + 1) \frac{dy}{dx} - y = e^{3x} (x + 1)^3$

251.

Ishan wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2. Using matrices, find the dimensions of the plot. Also give reason why he wants to donate the plot for a school.

252.

Given that vectors $\vec{a}, \vec{b}, \vec{c}$ form a triangle such that $\vec{a} = \vec{b} + \vec{c}$. Find p, q, r, s

such that area of triangle is $5\sqrt{6}$ where $\vec{a} = p\hat{i} + q\hat{j} + r\hat{k}$, $\vec{b} = s\hat{i} + 3\hat{j} + 4\hat{k}$ and

$\vec{c} = 3\hat{i} + \hat{j} - 2\hat{k}$.

253.

Find the equation of plane passing through the points A (3, 2, 1), B (4, 2, −2) and C (6, 5, −1) and hence find the value of λ for which A (3, 2, 1), B (4, 2, −2), C (6, 5, −1) and D (λ, 5, 5) are coplanar.

254.

OR

Find the co-ordinates of the point where the line $\vec{r} = (-\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 3\hat{k})$ meets the plane which is perpendicular to the vector $\vec{n} = \hat{i} + \hat{j} + 3\hat{k}$ and at a distance of $\frac{4}{\sqrt{11}}$ from origin.

255.

Let $f : N \to N$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : N \to S$ is invertible (where S is range of f). Find the inverse of f and hence find $f^{-1}(31)$ and $f^{-1}(87)$.

256.

Using properties of determinants, prove that :

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

257.

Find the maximum and minimum values of $f(x) = \sec x + \log \cos^2 x$, $0 < x < 2\pi$.

258.

Using integration find the area of the region $\{(x, y) : y^2 \le 6ax \text{ and } x^2 + y^2 \le 16a^2\}$

259.

Find the equation of the plane containing two parallel lines $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{3}$ and $\frac{x}{4} = \frac{y-2}{-2} = \frac{z+1}{6}$. Also, find if the plane thus obtained contains the line $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{5}$ or not.

260.

If $A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$, find α satisfying $0 < \alpha < \frac{\pi}{2}$ when $A + A^T = \sqrt{2}\, I_2$;

where A^T is transpose of A.

261.

Find the particular solution of differential equation : $\frac{dy}{dx} = -\frac{x + y\cos x}{1 + \sin x}$

given that $y = 1$ when $x = 0$.

262.

If $x\cos(a+y) = \cos y$ then prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$.

Hence show that $\sin a \frac{d^2y}{dx^2} + \sin 2(a+y)\frac{dy}{dx} = 0$.

263.

Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left[\frac{6x - 4\sqrt{1-4x^2}}{5}\right]$

264.

Let $A = R \times R$ and $*$ be a binary operation on A defined by

$(a, b) * (c, d) = (a + c, b + d)$

Show that * is commutative and associative. Find the identity element for * on A. Also find the inverse of every element $(a, b) \in A$.

265.

Solve for x : $\tan^{-1}\left(\frac{2-x}{2+x}\right) = \frac{1}{2}\tan^{-1}\frac{x}{2}, x > 0$.

266.

Solve the differential equation :

$$x\frac{dy}{dx} + y - x + xy\cot x = 0;\ x \neq 0.$$

267.

Find the maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$

268.

Matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric, find values of a and b.

269.

The two vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represent the two sides AB and AC, respectively of a ΔABC. Find the length of the median through A.

270.

The monthly incomes of Aryan and Babban are in the ratio 3 : 4 and their monthly expenditures are in the ratio 5 : 7. If each saves ₹ 15,000 per month, find their monthly incomes using matrix method. This problem reflects which value ?

271.

If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0.$

272.

Find the values of p and q, for which

$$f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x}, & \text{if } x < \frac{\pi}{2} \\ p, & \text{if } x = \pi/2 \\ \frac{q(1-\sin x)}{(\pi - 2x)^2}, & \text{if } x > \pi/2 \end{cases}$$

is continuous at $x = \pi/2$.

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273.

Evaluate $\int_{0}^{\pi} e^{2x} \cdot \sin\left(\frac{\pi}{4} + x\right) dx$

274.

Three persons A, B and C apply for a job of Manager in a Private Company. Chances of their selection (A, B and C) are in the ratio 1 : 2 : 4. The probabilities that A, B and C can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3 respectively. If the change does not take place, find the probability that it is due to the appointment of C.

275.

Prove that $\begin{vmatrix} yz - x^2 & zx - y^2 & xy - z^2 \\ zx - y^2 & xy - z^2 & yz - x^2 \\ xy - z^2 & yz - x^2 & zx - y^2 \end{vmatrix}$ is divisible by $(x + y + z)$, and hence find the quotient.

276.

Find the intervals in which $f(x) = \sin 3x - \cos 3x$, $0 < x < \pi$, is strictly increasing or strictly decreasing.

277.

Find the coordinate of the point P where the line through A(3, – 4, –5) and B (2, –3, 1) crosses the plane passing through three points L(2, 2, 1), M(3, 0, 1) and N(4, –1, 0). Also, find the ratio in which P divides the line segment AB.

278.

If $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = 8$, write the value of x.

279.

If $A = \begin{pmatrix} 3 & 5 \\ 7 & 9 \end{pmatrix}$ is written as A = P + Q, where P is a symmetric matrix and Q is skew symmetric matrix, then write the matrix P.

280.

If $\left|\vec{a} \times \vec{b}\right|^2 + \left|\vec{a} \cdot \vec{b}\right|^2 = 400$ and $\left|\vec{a}\right| = 5$, then write the value of $\left|\vec{b}\right|$.

281.

Write the equation of a plane which is at a distance of $5\sqrt{3}$ units from origin and the normal to which is equally inclined to coordinate axes.

282.

A coaching institute of English (subject) conducts classes in two batches I and II and fees for rich and poor children are different. In batch I, it has 20 poor and 5 rich children and total monthly collection is ₹ 9,000, whereas in batch II, it has 5 poor and 25 rich children and total monthly collection is ₹ 26,000. Using matrix method, find monthly fees paid by each child of two types. What values the coaching institute is inculcating in the society ?

283.

Find the values of a and b, if the function f defined by

$$f(x) = \begin{cases} x^2 + 3x + a, & x \leq 1 \\ bx + 2, & x > 1 \end{cases}$$

is differentiable at $x = 1$.

284.

Find the angle of intersection of the curves $y^2 = 4ax$ and $x^2 = 4by$.

285.

Evaluate : $\displaystyle\int_0^{\pi} \frac{x}{1 + \sin \alpha \sin x}\, dx$

286.

If f, g : R → R be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x$, $\forall x \in R$. Then find fog and gof. Hence find fog(–3), fog(5) and gof (–2).

287.

Using integration find the area of the region bounded by the curves $y = \sqrt{4 - x^2}$, $x^2 + y^2 - 4x = 0$ and the x-axis.

288.

Find the equation of the plane which contains the line of intersection of the planes $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ and whose x-intercept is twice its z-intercept.

Hence write the vector equation of a plane passing through the point (2, 3, –1) and parallel to the plane obtained above.

289.

Bag A contains 3 red and 5 black balls, while bag B contains 4 red and 4 black balls. Two balls are transferred at random from bag A to bag B and then a ball is drawn from bag B at random. If the ball drawn from bag B is found to be red, find the probability that two red balls were transferred from A to B.

290.

In order to supplement daily diet, a person wishes to take X and Y tablets. The contents (in milligrams per tablet) of iron, calcium and vitamins in X and Y are given as below :

Tablets	Iron	Calcium	Vitamin
X	6	3	2
Y	2	3	4

The person needs to supplement at least 18 milligrams of iron, 21 milligrams of calcium and 16 milligrams of vitamins. The price of each tablet of X and Y is ₹ 2 and ₹ 1 respectively. How many tablets of each type should the person take in order to satisfy the above requirement at the minimum cost ? Make an LPP and solve graphically.

291.

If A and B are invertible matrices of order 3, $|A| = 2$ and $|(AB)^{-1}| = -\frac{1}{6}$. Find $|B|$.

292.

Write the order of the differential equation:

$$\log\left(\frac{d^2y}{dx^2}\right) = \left(\frac{dy}{dx}\right)^3 + x$$

293.

Find the direction cosines of the line: $\frac{x-1}{2} = -y = \frac{z+1}{2}$

294.

Let $A = Z \times Z$ and * be a binary operation on A defined by

$(a, b) * (c, d) = (ad + bc, bd)$.

Find the identity element for * in the set A.

295.

Find : $\int \frac{(x^2 + \sin^2 x)\sec^2 x}{1 + x^2}\, dx$

296.

Using properties of determinants, prove that:

$$\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$$

297.

If $y = \log(1+2t^2+t^4)$, $x = \tan^{-1} t$, find $\dfrac{d^2y}{dx^2}$

298.

If $y = \cos(m\cos^{-1} x)$

Show that: $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + m^2y = 0$

299.

. Find: $\displaystyle\int \frac{x^4+1}{x(x^2+1)^2}dx$

300.

Evaluate: $\displaystyle\int_{-1}^{1} \frac{x+|x|+1}{x^2+2|x|+1}dx$

301.

Find the particular solution of the following differential equation.

$\cos y\, dx + (1+2e^{-x})\sin y\, dy = 0;\ y(0) = \dfrac{\pi}{4}$

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302.

Find the general solution of the differential equation:

$$\frac{dx}{dy}=\frac{y\tan y - x\tan y - xy}{y\tan y}$$

303.

If $\vec{p}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{q}=\hat{i}-2\hat{j}+\hat{k}$, find a vector of magnitude $5\sqrt{3}$ units perpendicular to the vector $\vec{q}$ and coplanar with vectors $\vec{p}$ and $\vec{q}$.

304.

Find the vector equation of the line joining (1, 2, 3) and (–3, 4, 3) and show that it is perpendicular to the z-axis.

305.

A cuboidal shaped godown with square base is to be constructed. Three times as much cost per square meter is incurred for constructing the roof as compared to the walls. Find the dimensions of the godown if it is to enclose a given volume and minimize the cost of constructing the roof and the walls.

306.

Find the area bounded by the curves $y=\sqrt{x}$, $2y+3=x$ and x – axis.

307.

Find the area of the region.

$$\{(x, y) : x^2+y^2 \le 8,\ x^2 \le 2y\}$$

308.

Find the equation of the plane through the line $\frac{x-1}{3}=\frac{y-4}{2}=\frac{z-4}{-2}$ and parallel to the line $\frac{x+1}{2}=\frac{1-y}{4}=\frac{z+2}{1}$. Hence, find the shortest distance between the lines.

309.

Show that the line of intersection of the planes x + 2y + 3z = 8 and 2x + 3y + 4z = 11 is coplanar with the line $\frac{x+1}{1}=\frac{y+1}{2}=\frac{z+1}{3}$. Also find the equation of the plane containing them.

310.

The members of a consulting firm rent cars from three rental agencies:
50% from agency X, 30% from agency Y and 20% from agency Z.
From past experience it is known that 9% of the cars from agency X need a service and tuning before renting, 12% of the cars from agency Y need a service and tuning before renting and 10% of the cars from agency Z need a service and tuning before renting. If the rental car delivered to the firm needs service and tuning, find the probability that agency Z is not to be blamed.

311. If A is an invertible matrix of order 2 and $det(A) = 4$, then write the value of $det(A^{-1})$.

312. Evaluate: $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\, dx$

313. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\jmath} - \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$.

314. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $|\vec{c}| = 7$ show that angle between $\vec{a}$ and $\vec{b}$ is $60°$.

315. Evaluate: $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}\, dx$

316. Find the equation of the plane passing through the points $(3, 4, 1)$ and $(0, 1, 0)$ and parallel to the line $\frac{x+3}{2} = \frac{y-3}{7} = \frac{z-2}{5}$.

317. Show that height of the cylinder of maximum volume that can be inscribed in a cone of height h is $\frac{1}{3}h$.

318. Find the point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance $3\sqrt{2}$ from the point $(1, 2, 3)$.

319. Evaluate: $\int \frac{(x-4)e^x}{(x-2)^3} dx$

320. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ show that $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$ where $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$.

321. Find the particular solution, satisfying the given condition for the following differential equation $\frac{dy}{dx} - \frac{y}{x} + \text{cosec}\left(\frac{y}{x}\right) = 0$; $y = 0, when\ x = 1$.

322. Evaluate: $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} - e^{-\cos x}} dx$

By **Arun Kumar Shukla**

Answers

1. -1
2. $\frac{2}{3}$
3. -1
5. $\frac{11}{26}$
7. $\frac{\pi}{3}$
8. $\frac{\pi}{4(1+x^2)}$
9. 7
10. $\frac{23}{6}$
11. $(4,-3,-2)$
12. $A=2, B=3, Profit=Rs.230$
13. $a=-2, b=3$
14. 3
15. $-\frac{1}{2}$
18. $\frac{e^x}{x}$
19. $Rs.3500, Rs.2500$
20. $\frac{1}{\sqrt{3}}$
21. $\left(-\frac{5}{3},\frac{2}{3},\frac{19}{3}\right)$
22. $\cos^{-1}\left(\frac{2}{3}\right), \cos^{-1}\left(\frac{1}{3}\right), \cos^{-1}\left(\frac{2}{3}\right)$
24. π
25. $y-(a\sin\theta - a\theta\cos\theta) = \tan\theta\,(x - a\cos\theta - a\theta\sin\theta)$
    $y\sin\theta + x\cos\theta = a$
27. $(2,-1,2)$
28. 0.18
30. $y = e^{\frac{x}{y}} + C$
31. $(2,6,-2)$
32. $k=\frac{1}{2\sqrt{2}}$ or $\frac{\sqrt{2}}{4}$
33. $\frac{200}{231}$
34. $\left(\frac{\pi}{2}-\frac{4}{3}\right)a^2$
35. $\{(1,3),(2,4)\}$
36. -15
37. $\frac{1}{2}$
38. $\begin{bmatrix}3 & 2\\5 & 3\end{bmatrix}, P=\begin{bmatrix}13 & 8\\1 & 1\end{bmatrix}$
40. 0.0005
41. $\frac{1}{2}e^x\sec x + C$
43. 16
44. $a=1, a=2$
45. $a=0, b\in R-\{0\}$
47. $y=-3x+3, y=7x-14$
48. $Strictly\ decreasing\ in\ (-1,0), strictly\ increasing\ in\ (0,\infty)$
49. $20\ trees, Rs.2000$
50. $\frac{1}{4}\log|1+\sin x| + \frac{1}{2}\left(\frac{1}{1+\sin x}\right) - \frac{1}{4}\log|1-\sin x| + C$
51. $x=-y^2e^{-y}+\frac{y^2}{e}$
53. $\frac{x-1}{3}=\frac{y-1}{10}=\frac{z-1}{17}$
54. $\frac{64}{199}$
55. $\sqrt[3]{\frac{x-7}{2}}, 1$
56. 0
57. $x=1, y=1, z=1$
58. $[0 \quad 1 \quad 0]$
59. $\left(\frac{1}{6}+\frac{\pi}{4}\right)$
60. $\frac{\sqrt{3}\pi^2}{18}$
61. $200, 120, profit\ Rs.2412$
62. 162
63. $\sqrt{q^2+r^2}$

64. $\frac{3x^2-5}{9x^4-30x^2+26}$
65. 2
67. 5
68. 0
69. $a = 11, b$ is arbitrary
70. $n = 10$
71. $-\frac{1}{3}\log|\tan x + 1| + \frac{1}{6}\log|\tan^2 x - \tan x + 1| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x - 1}{\sqrt{3}}\right) + C$
72. Identity and inverse does not exists.
73. $x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{6}}, z = \pm\frac{1}{\sqrt{3}}$
74. $e^{\sin^2 x}\cos^{-1}x\left[\sin 2x - \frac{1}{\cos^{-1}(x\sqrt{1-x^2})}\right]$
75. $\begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & -2 \\ 3 & 2 & 0 \end{bmatrix}$
76. 4%
77. 5
79. $a = 4, b = -4$
80. $-\frac{1}{6}\log|1 + \cos x| - \frac{1}{2}$
81. π^2
82. $\frac{13}{15}, \frac{5}{18}$
83. $1\ sq.units$
84. $(-5, 10, -3), 4\sqrt{14}, \left(-\frac{3}{7}, -\frac{22}{7}, \frac{15}{7}\right)$
85. $\frac{20}{37}$
86. $\log|x| - \log|x + \sin x| + C$
87. $x + \frac{1}{4}\log|x^2 + 1| + \frac{1}{2}\log|x - 1| - \frac{1}{2}\tan^{-1}x + C$
88. $\frac{\pi}{6}$
89. $\frac{1}{4}\log 3$
90. $\frac{72}{125}$
91. $4\sqrt{2}units$
92. $m = 2$
93. $\frac{6}{\sqrt{34}}$
94. $\hat{\imath} - 11\hat{\jmath} - 7\hat{k}$
95. $\frac{\pi}{2}$
96. 1
98. $24600, 15800, 576, 332$
99. $\frac{17}{7}$
100. 3
101. $1 + y^2$
102. $\sqrt{6}$
103. $\frac{6}{7}, \frac{2}{7}, -\frac{3}{7}\ or\ -\frac{6}{7}, -\frac{2}{7}, \frac{3}{7}$
104. $Rs. 30{,}000; Rs. 23000; Rs. 39000$
105. $\frac{2}{\sqrt{3}}$
106. $\frac{6}{5}$
107. $-\frac{\log x}{x+1} + \log\left(\frac{x}{x+1}\right) + C$
108. $\frac{1}{4}$
109.

x	0	1	2	3
$P(x)$	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

111. $e = 0, \frac{-a}{1+a}$
112. $0, 4$
113. $80days, 60days, cost = Rs. 1860{,}000$
114. 4
115. $\pm 2\sqrt{10}$
116. $2a^2$
117. 10%
118. $No\ any\ point\ of\ discountinuity$
120. 2π
121. $x\left[\log(\log x) - \frac{1}{\log x}\right] + C$
122. $\log|\cos x + x\sin x| + C$

123. $\frac{x^2}{2} + x + \frac{1}{2}\log|x-1| - \frac{1}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1}x + C$

124. $\frac{-10\hat{\imath}-7\hat{\jmath}+4\hat{k}}{\sqrt{165}}$ or $\frac{10\hat{\imath}+7\hat{\jmath}-4\hat{k}}{\sqrt{165}}$

125. 2

126. $bcx + acy - abz = 0$

127. $\frac{1}{4}$

129. $(\pi - 2) sq.units$

130. $9\sqrt{3}\ sq.units$

131. $\sqrt{3}e$

132. ± 12

136. $-\frac{1}{2}\left(1+\frac{1}{x^5}\right)^{\frac{2}{5}} + C$

137. $Rs. 50500, Rs. 40800, Rs. 41600$

138. $\{(-1,2),(0,0),(1,0),(2,2)\}$

139. $Absolute\ Max.: \frac{5}{4}\ at\ \frac{\pi}{2}\ and\ \frac{5\pi}{6}. Absolute\ min.: 1\ at\ 0, \frac{\pi}{6}\ and\ \pi$

143. $\frac{3}{4}$

144. $\frac{\pi}{4}$

145. $\frac{5}{2\pi} - \frac{1}{\pi^2}$

146. $x - \log x + \frac{1}{2}\log|x^2+1| - \tan^{-1}x + C$

147. $\sin^{-1}\left(\frac{4}{3\sqrt{10}}\right)$

148. 2

149. $y = x\frac{dy}{dx} + \sqrt{1+\left(\frac{dy}{dx}\right)^2}$

150. $1, 1, \pm\sqrt{2}\ ; x + y \pm \sqrt{2}z - 1 = 0$

151. $Minimum: 2150\ at\ (6,4)$

152. $\frac{d^2y}{dx^2} + a^2y = 0$

153. $\frac{3}{13}, \frac{4}{13}, \frac{12}{13}$

154. $A = Rs. 1050, B = Rs. 1150, C = Rs. 1300$

155. 1

156. $\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

158. $x^{e^{-x^2}} e^{-x^2}\left(\frac{1}{x} - 2x\log x\right)$

160. $\log|\sec x + \tan x| - 2\tan\frac{x}{2} + C$

161. $\frac{x^2}{2}\sin^{-1}x + \frac{x}{4}\sqrt{1-x^2} - \frac{1}{4}\sin^{-1}x + C$

162. $\frac{8}{3} + \frac{e^5 - e}{2}$

163. $\frac{\pi^2}{4}$

164. $(4, 0, -1)$

165. $\frac{1}{4}$

166. $x - 3y + 2z + 3 = 0;\ \vec{r}.(\hat{\imath} - 3\hat{\jmath} + 2\hat{k}) + 3 = 0$

167. $\frac{1}{5}$

169. $4\ sq.units$

170. $(0,0)$

171. $y = \tan x - \sqrt{\tan x}$

172. $Max.: Rs. 12500\ at\ x = 10, y = 50$

173. $ax(3x + 2a)$

174. $\left(2\pi + \frac{1}{2a}\sin 2a\pi - \frac{\sin 2b\pi}{2b}\right)\ or\ 2\pi$

175. $\frac{22}{45}$

176. 0

177. $-\frac{1}{2}$

178. -1

181. $2\sqrt{3}\ sq.units$

182. $\frac{-e^{-1}(e^{-6}-1)}{3} + \frac{32}{3}$

183. $x = \tan^{-1}y - 1 + ce^{-\tan^{-1}y}$

184. $5x - 2y - z - 6 = 0$

185. $P(A) = \frac{1}{5}, P(B) = \frac{1}{6}$ or $P(A) = \frac{5}{6}, P(B) = \frac{4}{5}$

186. Local maximum Value $= \sqrt{2}$; Local Minimum value $= -\sqrt{2}$

187. $r^2 \frac{d^2v}{dr^2} + 2r\frac{dv}{dr} = 0$

188. $\frac{2^y}{\log 2} = x + C$

189. $-\frac{4}{5}\log|x^2+4| + \frac{9}{5}\log|x^2+9| + C$

190. 1

191. $\frac{17}{400}$

192. $\frac{\sqrt{34}}{2}$

193. $x - y + z - 1 = 0$

194. $\theta = \frac{n}{n+2}$

195. Not differentiable at $x = 1$, Differentiable at $x = 2$

196. $\frac{4}{13}\left[\frac{e^{2x}}{2}\left(-\frac{3}{2}\cos(3x+1) + \sin(3x+1)\right)\right] + C$

197. $f^{-1}(y) = \left(\frac{\sqrt{y-6}-3}{2}\right)$

198. $\frac{10}{3}$ sq. units

199. $2\sqrt{abc}$

200. $(-2, -8)$

By **Arun Kumar Shukla**

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